

# **ACOUSTIC NOISE CONTROL USING MULTIPLE EXPANSION CHAMBERS**

BY

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A Thesis Presented to the  
DEANSHIP OF GRADUATE STUDIES

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the  
Requirements for the Degree of

**MASTER OF SCIENCE**

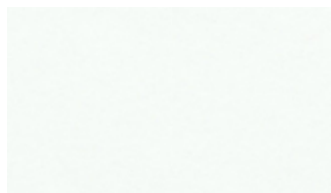
In

**MECHANICAL ENGINEERING**

**December, 2011**



**In the name of Allah, the Most Gracious and the  
Most Merciful**



**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**  
**DHAHRAN 31261, SAUDI ARABIA**

**DEANSHIP OF GRADUATE STUDIES**

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*Dedicated*

*to*

*My Beloved Parents, Brothers, My wife*

*And my son*

## **ACKNOWLEDGMENTS**

All praise and thanks are due to Almighty Allah, Most Gracious and Most Merciful, for his immense beneficence and blessings. He bestowed upon me health, knowledge and patience to complete this work. May peace and blessings be upon prophet Muhammad (PBUH), his family and his companions.

Thereafter, acknowledgement is due to KFUPM for the support extended towards my research through its remarkable facilities and for granting me the opportunity to pursue graduate studies.

I acknowledge, with deep gratitude and appreciation, the inspiration, encouragement, valuable time and continuous guidance given to me by my thesis advisor, Dr. Muhammad Hawwa. I am highly grateful to my Committee members, Dr. Yagoub N. Al-Nassar and Dr. Hussain M. Al-Qhtani for their constructive guidance and support.

My heartfelt thanks are due to my parents, brothers, my son, and my wife for their prayers, guidance, and moral support throughout my academic life. My parents' advice, to strive for excellence has made all this work possible.

Special thanks are due to my senior colleagues at the university, for their help, prayers and who provided wonderful company and good memories that will last a life time.

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## **THESIS ABSTRACT (ENGLISH)**

**NAME:** MAMON HOROB

**TITLE:** ACOUSTIC NOISE CONTROL USING MULTIPLE  
EXPANSION CHAMBERS

**MAJOR FIELD:** MECHANICAL ENGINEERING

**DATE OF DEGREE:** 1433 (H) (DEC 2011 G)

Silencers with expansion chambers are often used to reduce noise in different applications such as HVAC, exhaust systems, and car muffler systems. This study focuses on the effectiveness of multiple expansion chambers in reducing noise. Methods for enhancing silencer designs are realized by utilizing extended tubes, changing expansion chamber cross sectional areas and changing the lengths of the expansion chambers and connecting tubes. General guidelines for design enhancement are provided through this study.

**MASTER OF SCIENCE DEGREE**

**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**

Dhahran, Saudi Arabia

## THESIS ABSTRACT (ARABIC)

### ملخص الرسالة

الاسم: مأمون محمد عبدالقادر حروب

عنوان الرسالة: السيطرة على الضوضاء باستخدام الحجيرات التوسعية المتعددة.

التخصص: الهندسة الميكانيكية

تاريخ التخرج: 1433 هـ - (ديسمبر 2011 م)

حجيرات التوسع تستخدم غالبا للحد من الضوضاء الصوتية في نظم مختلفة كأنظمة التكييف و كتم الصوت في أنظمة الرفع و في المركبات. وترتكز هذه الدراسة على فعالية الحجيرات التوسعية المتعددة وذلك من اجل الحد من الضوضاء في هذه النظم. التصميم المثالي لحجيرات التوسع تتحقق من خلال الاستفادة من التغير في مساحات حجيرات التوسع , التغير في اطوال حجيرات التوسع , التغير في المساحات والاطوال معا على حد سواء.

شهادة ماجستير علوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

# CHAPTER1

## INTRODUCTION

Acoustics is the science that deals with the properties of sound, like amplitude, frequency, complexity of sound waves and how the sound spreads through the environment [1]. Acoustics includes a vast range of topics, including: noise control, SONAR for submarine navigation, ultrasounds for medical imaging, thermo acoustic refrigeration, seismology, bioacoustics, and electro acoustic communication[2]. There are many types of sound in our life, some of them are useful and the others are noisy, as illustrated in the following Figure.

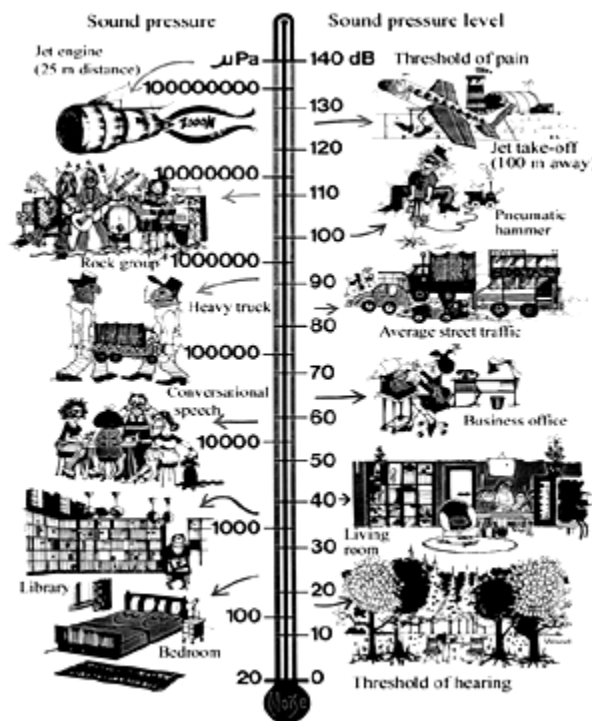


Figure 1.1: Sound Pressure Level (SPL) Examples [3]

Sound spread in the environment as a wave. A sound wave is described in terms of magnitude, frequency, sound velocity, and wave number. The relationship between sound speed and frequency is given by

$$c = \lambda f \quad (1.1)$$

The velocity of sound depends on the environment which the sound travels through. As an example, the value of sound velocity in air equals 330 m/s at standard pressure.

Human ears hear sounds that have a frequency range from 20 Hz to 20 kHz. In daily life the sound is usually a mixture of several acoustic waves with different frequencies. Normal sound becomes noise when it is annoying depending on the amplitude of the sound wave and its frequency. The human ear can detect a wide range of sound levels and frequencies, but it is more sensitive to some frequencies than others.

There are several measures of the magnitude of sound: sound pressure level and sound power level. Sound pressure is a sound property at a given location, sound power level is the rate at which acoustical energy is radiated from a sound source and its unit is the watt; it is a property of the sound source and it gives the total acoustic power emitted by the sound source and can be measured by a microphone. Sound intensity (I) is another measure of sound, and it can be calculated.

$$I = \frac{W}{A} \left[ \frac{\text{watt}}{\text{m}^2} \right] \quad (1.2)$$

Where  $W$  = acoustical sound power of the source (watt)

$A$  = surface area ( $\text{m}^2$ )



Acoustic signals are measured with decibel (dB), which is a logarithmic unit that indicates the ratio of relative to a specified reference level of sound pressure, power, or intensity. The gains of sound amplifiers, attenuation of acoustic signals, and signal-to-noise ratios are often expressed in decibels.

## **1.1 SILENCERS**

Mufflers or acoustic silencers are devices used to reduce severity of the sound waves (noise) propagated in the environment. Typical applications include exhaust systems, car muffler systems, air handling systems, ducted pumps, air discharge lines and compressors.

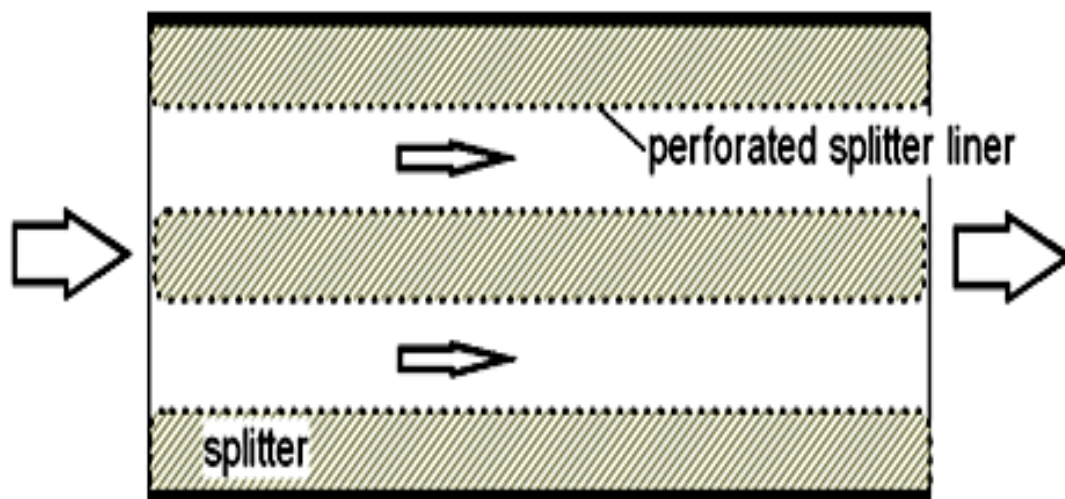
There are four types of commonly used silencers. These are[4]:

- 1) Absorptive or dissipative silencers.
- 2) Reactive or reflective silencers.
- 3) Dispersive or diffusive silencers.
- 4) Active silencers.

### **1.1.1 Absorptive or Dissipative Silencers**

Absorptive silencers attenuate the sound waves (noise) depending on a material used to absorb the sound waves. Absorptive silencer is mostly used in HVAC system.

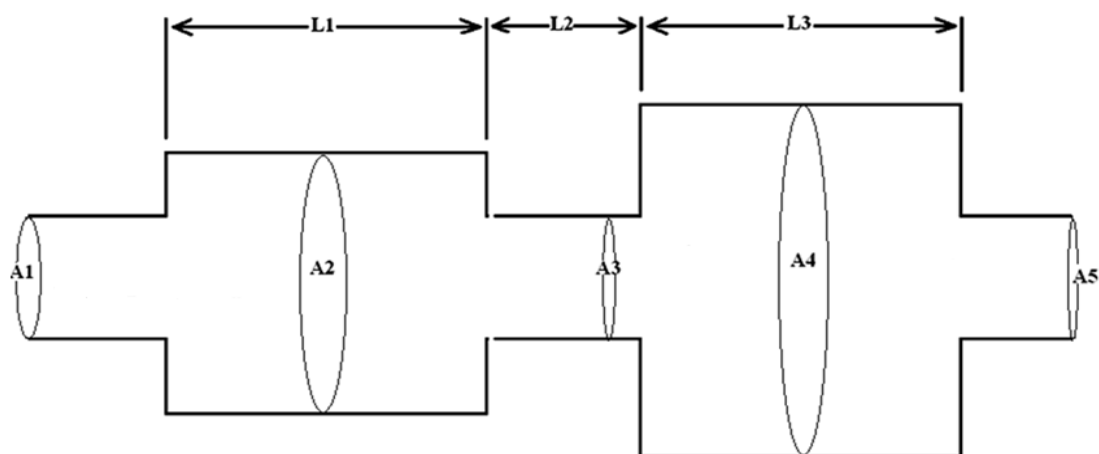
The thickness of acoustical material selected depends on the frequency of the noise. Some of the incident sound power is converted to heat caused due to the sound motion inside the acoustical material. Figure 1.2 shows schematic example of absorptive silencer [4].



**Figure1.2:An example of absorptive silencer.**

### **1.1.2 Reflective or Reactive Silencers**

Reactive silencer function is by reflecting sound back to the source. The sound energy is dissipated due to the internal reflection inside the flow path and by absorption at the source. Reactive or reflective silencers have tuned membranes and cavities and are designed to absorb machine noise having low frequencies. Figure 1.3 shows an example of reactive silencer.



**Figure1.3: Double expansion chambers as an example of reactive silencer.**

### 1.1.3 Diffuser or Depressive Silencers

Diffuser silencer has a perforated wall which slows down the flow velocity of the sound. It is mainly used in applications which contain control valve and nozzle. Figure 1.4 shows an example of diffuser silencer [5].



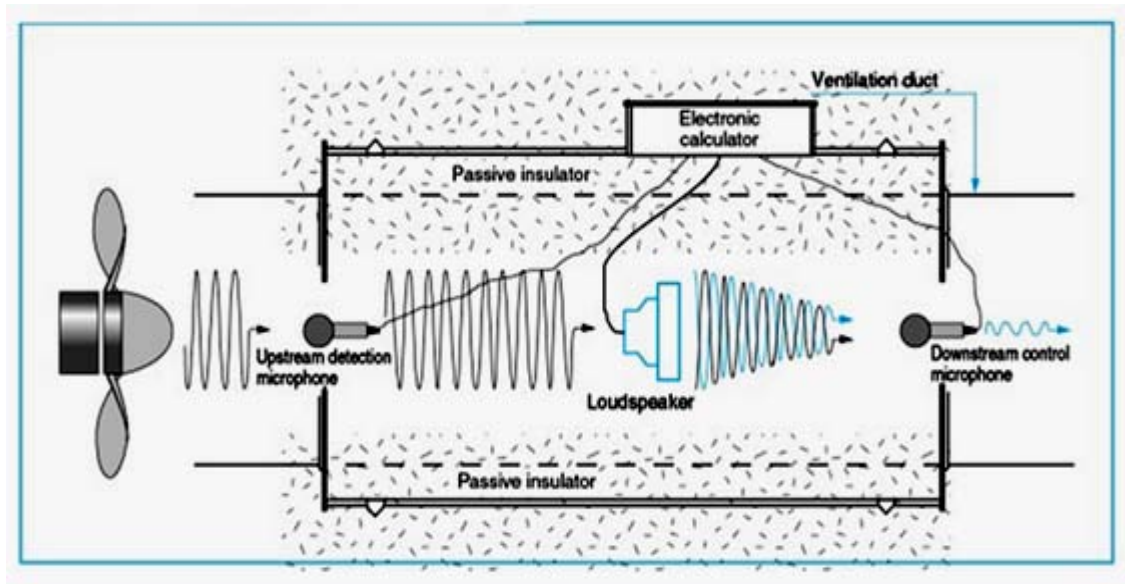
**Figure 1.4: An example of diffuser silencer.**

### 1.1.4 Active Silencers

Active noise control has new modification sound field. This modification uses speaker, microphone and electronics device to determine and attenuate or cancel the sound wave (noise).

In its simplest form, a control system uses a microphone to get the noise wave. Then by using a speaker, the control system produces a sound wave has same frequency and amplitude of noise wave but in the opposite direction (like mirror-image of the noise

wave). The speaker thus eliminates the noise wave, and the net result is no sound at all. Active noise control is mostly used for applications having steady noise wave like engines, fans, etc. Figure 1.5 shows schematic example of Active silencer [6].



**Figure 1.5: An example of active silencer.**

## 1.2 EXPANSION CHAMBERS

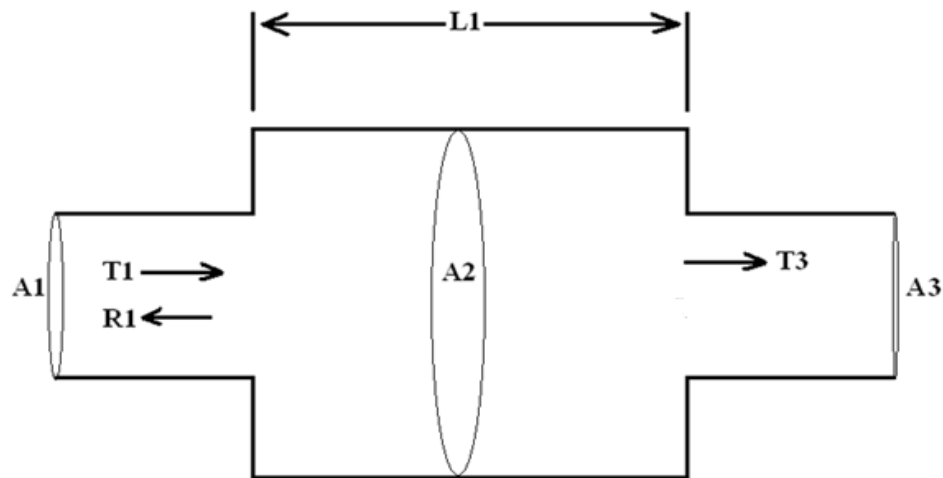
Silencers are important control devices for noise reduction. In general, the expansion-chamber silencer is one of the main types of reactive silencers, which are useful devices used to attenuate the acoustic noise in a frequency range of acoustic field generated by mechanical systems, where noise is reduced by reflection of sound energy within a duct.

Reactive silencers reduce acoustical noise through the existence of acoustical impedance discontinuity which leads to reflecting noise waves back to the source. The discontinuity is achieved by:

- (a) Sudden cross-sectional area change (expansions),
- (b) Wall properties change,
- (c) Or combination of (a) and (b).

Expansion chambers are mostly used in heating ventilation and air conditioning (HVAC) systems, exhaust systems, and car muffler systems. The expansion chamber mufflers have one or more than one expansion chamber or expansion volume to act as silencers.

Figure 1.6 shows reactive silencer which has single expansion chamber with length  $L_1$  and cross sectional area  $A_2$ . The most familiar example of a single expansion chamber is probably the automotive muffler as Figure 1.7 shows.

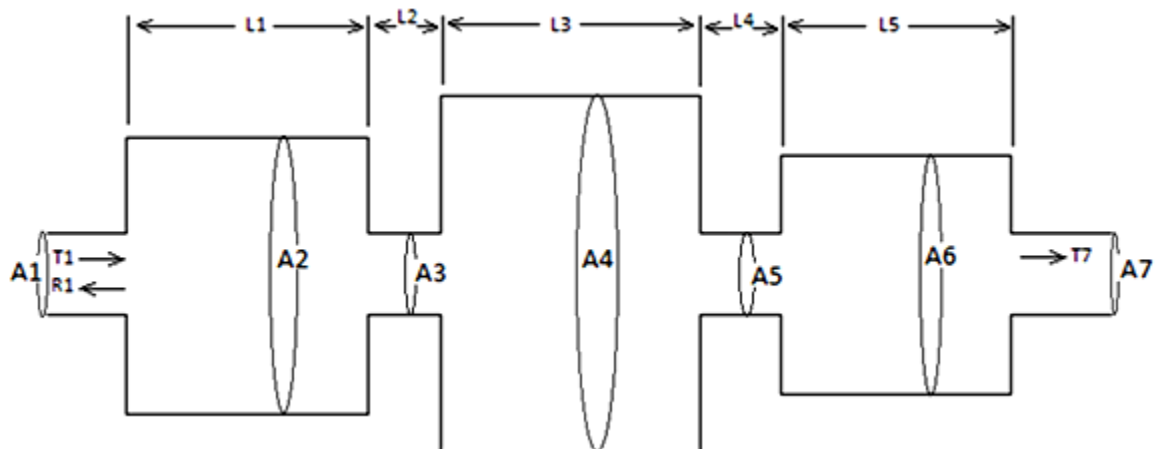


**Figure 1.6: Geometry of single expansion chamber silencer where  $A_j$  are the cross-sectional areas and  $L$  is the length.**



**Figure 1.6: Car muffler as example of single expansion chamber reactive silencer.**

To increase the attenuation of acoustics noise, most manufacturers of reactive silencers used more than one expansion chamber in series. Figure 1.8 shows a reactive silencer having multiple expansion chambers (triple expansion chambers) with different lengths and cross sectional areas, the values of lengths and the areas depend on the frequency and the amplitude of the sound wave that need to be reduced.



**Figure 1.8: Geometry of triple expansion chamber with unequal volume expansion chambers where  $A_j$  : are the cross-sectional areas and  $L_j$  : are the lengths.**

### 1.3 LITERATURE REVIEW

Mufflers with multiple expansion chambers were considered by a number of researchers. Lamancusa[7] considered sound attenuation due to double expansion chamber and derived the transmission loss coefficients for various cases of equal and unequal expansion chambers and he calculated the transmission loss of double expansion chamber with different geometry by changing randomly two parameters, the areas of the double expansion chambers and the lengths of the double expansion chamber silencer. He concluded that the amplitude and width of the stopband is changed with the connecting tube length. Selamet and Radavich[8] studied the effect of changing the length of a concentric expansion chamber. Three approaches are employed to determine the transmission loss: (1) a two-dimensional, axisymmetric analytical solution; (2) a three-dimensional computational solution based on the boundary element method; and (3) experiments on an extended impedance tube set-up with nine expansion chambers fabricated with fixed inlet and outlet ducts. The results from all three approaches agreed well.

Selamet and Ji[9] developed a three dimensional analytical approach used to calculate the acoustical performance (transmission loss) of a circular expansion chamber which has inlet and outlet offset ducts. They compared the three dimensional result with the one dimensional prediction, they conclude that the results from all approaches agreed well. Tao and Seybert[10] studied the transmission loss for two muffler types with single and double expansion chamber by the two load method and the two source method. The measured transmission losses compared well for both cases, and showed that the transmission loss can be measured by these methods without an anechoic termination.

Suwandi et al.[11] used the plane wave transmission line theory to evaluate the acoustic performance (Transmission Loss) of various mufflers with single expansion chambers and mufflers containing porous material with extended inlet and outlet pipes. They also performed experimental measurements and showed that experimental results agreed well with the theoretical results. Gerges et al.[12] summarized the transfer matrix principle for calculating the transmission loss of a muffler with single expansion chamber and they applied this method to more than one expansion chamber configurations. They did experimental measurements and verified numerical solution. They also concluded that the effects of extending entrance tube was to increase the transmission loss peaks and to expand the frequency bands of attenuation.

Ji[13] studied the acoustic attenuation performance (Transmission Loss) of multi-expansion chambers reactive silencer with inter-connecting tubes using the boundary element method. He compared transmission loss results of mufflers having single, double and triple expansion chambers. He concluded that using the multiple expansion chamber silencers in attenuating sound is better than using a single one. Barbieri[14] described a new methodology which collaborated the finite element analysis and Zoutendijk's feasible directions method and they studied the transmission loss affected by changing the geometry dimensions of muffler with single expansion chamber.

Wu et al. [15] presented the acoustic attenuation performance (Transmission loss) of different silencer configurations, a case with single-inlet-double-outlets and a case of double-inlets-single outlet rectangular expansion chambers. They used three different methods to measure transmission through these expansion chambers to verify accuracy



and to study the effect of the higher order modes. They later in [16] used a single-inlet-double-outlet cylindrical expansion chamber muffler to calculate transmission loss by using a modal meshing approach and a finite element method and compared the results of the two methods.

Masson et al.[17] optimized the acoustical performance of low cost and simple geometry expansion chamber silencer by using micro perforated panels. They computed the transmission loss by using the transfer matrix method and the boundary element method. Different muffler configurations based on micro perforated panels in the expansion chambers were tested experimentally and the results were compared with theoretical ones.

Andersen [18] presented the transmission loss of different configurations of silencers which were the reactive muffler, the absorptive muffler and the plug flow muffler. He used the transfer matrix method to calculate the transmission loss theoretically and did experiments to assure experimental results agreement with the theoretical ones. Kang and Ji[19] studied the transmission loss of Helmholtz extended tube and simple expansion chamber with extended inlet/outlet using the 1D analytical method.

Venkatesham et al.[20] analyzed the transmission loss of different configurations of rectangular expansion chambers with different location of inlet-outlet by means of Green's functions. They used the transfer matrix method in their analysis and compared the results with those obtained from finite element method. They concluded that the transfer matrix method was more efficient than the corresponding numerical method.

The other study that presented the expansion chamber silencer with perforated, absorptive material, and main flow be as following. Maa[21] studied the potential of a silencer with a microperforated panel. Wu et al.[22] measured the transmission loss of mufflers by using boundary element method and the four pole method. They improved the four pole method results by the numerical results. Kim and Ih[23] studied the sound attenuation due to an expansion chamber with curved duct bends using transfer matrix method.

Selamet et al. [24] used the one-dimensional analytical and three-dimensional boundary element methods to predict the acoustic attenuation of hybrid silencers without mean flow. Joseph et al. [25] investigated the relation between the sound power and acoustic pressure for a multi-mode sound transmission in ducts having flow. Dokumaci[26] studied the acoustics performance of mufflers with perforated pipe and how it is affected by sheared grazing mean flow. Kar and Munjal[27] generalized algebraic algorithm of a mufflers with different number of interacting ducts to produce the system matrix easily rather than using governing equation.

Zhao et al. [28] studied the characteristics of exhaust noise of pneumatic frictional clutch and brake. They found that the expansion chamber volume was an important parameter design to reduce the exhaust resistance. Rammal and Boden[29] used the two load method to study the characteristics of nonlinear acoustic sources. Their simulated results were compared well with experimental data from various sources. Broatch et al. [30] studied the transmission loss properties of different configuration of silencers using one dimensional gas flow equation and the mesh spacing.

Several effects on perforated dissipative mufflers were examined by Denia et al [31] such as the extended inlet-outlet ducts, absorbent resistivity, the porosity of the perforations, and the muffler dimensions. Wang et al. [32] studied the acoustical properties of standing wave duct system using different experimental methods. Chiu [33] optimized the multi-expansion chamber muffler shape with plug-inlet tube using genetic algorithms (GA) as well as a numerical decoupling.

Chiu and Chang [34] used transfer matrix method to calculate transmission loss of multi-expansion chamber perforated mufflers. The decoupling technique and plane wave theory were used [35] to solve the coupled acoustical problem of plug mufflers with perforated tubes.

Ji [36] investigated the effect of internal structure of hybrid expansion chamber silencer with perforated facing on the transmission loss using the substructure boundary element method. Chaitanya and Munjal [37] investigated theoretically the effect of wall thickness on the end corrections of the extended inlet and outlet of a double-tuned expansion chamber. Mimani and Munjal [38] studied the acoustical performance of different muffler configurations using transverse plane wave and matrix approaches.

## **1.4 MOTIVATIONS AND OBJECTIVES**

From time to time during the life, you hear a disturbing sound from different systems such as HVAC, exhaust systems, factories machines, car muffler systems and others. The potential for health hazards from these systems has been recognized. However, from this point it has gained momentum relatively recently as people became more and more

conscious of their working and living environment. Also, the governments of many countries have responded to popular demand with mandatory restrictions on sound emitted by disturbing systems.

Due to the importance and seriousness of this issue and to the decrease the human health hazards, the silencer was exclusively created to reduce the noise arising from the disturbing systems. This was created by a significant engineering exploit. Likewise, this thesis also utilized to studying the enhance improvement and effectiveness of the silencers representative by expansion chamber silencers.

It is clear from the review of the latest work on the analysis of expansion chamber silencer that most of the works conducted consider a specific geometry of the expansion chamber silencer because it is related to an experiment work. So there is no extensive study done to figure out or to suggest how to design an expansion chamber silencer that gives high acoustic performance at any wave noise need to attenuate.

Moreover, according to the literature considered in this thesis, there is no study focuses on the effectiveness of expansion chamber silencer with respect to increase the number of expansion chambers, changing cross sectional area of the multiple expansion chamber silencer, changing the lengths of the expansion chambers, changing the lengths of the connecting tubes, and changing the lengths of the extended tubes.

This thesis Focus was placed on silencers with two and three expansion chambers, as more number of expansion chambers can be economically unfeasible.

The general objective of this work is to compare the performance of a single and multiple expansion chambers as means of acoustic attenuation. Silencers with multiple expansion

chambers are allowed to have extended tubes, different lengths, and different cross sectional areas.

So, this study of the acoustic attenuation of multiple expansion chamber silencers will be carried out through the following:

- Changing the cross sectional areas of the expansion chambers.
- Changing the lengths of the expansion chambers.
- Allowing extend tubes of expansion chambers to have different lengths.

## 1.5 THESIS ORGANIZATION

This thesis is divided into six chapters as follows:

- **Chapter 1: Introduction:** This chapter introduces the topic area investigated in the thesis. It provides an introduction about the topic, followed by literature view on topic related to expansion chamber silencer, then research objectives of the thesis.
- **Chapter 2: Mathematical Modeling:** This chapter provides the general equations and the method used to found the mathematical model of single expansion chamber silencer.
- **Chapter 3: Analysis of Silencers Having a Multiple expansion chambers:** In this chapter presents the analysis of acoustic attenuation due to double and triple expansion chamber silencers. Parametric study is carried out on the change of

transmission loss as a function of investigated the area ratio and the length of the expansion chambers.

- **Chapter 4: Enhancing Attenuation (I) Introducing Extended Tubes:** In this chapter, mathematical formulation of expansion chamber silencer with extended tube is discussed using transfer matrix method. Also, investigate the difference between expansion chamber silencer with and without extended tubes.
- **Chapter 5: Enhancing Attenuation (II) Applying Chirp and Taper Functions:** This chapter discussed how to design expansion chamber silencer geometry by utilizing modifications in cross sectional areas or modifications in the lengths of the expansion chambers and connecting tubes, or combined changes of both. This modification carried out by applying taper and chirp functions to the expansion chamber geometry
- **Chapter 6: Conclusions and Recommendations:** This chapter provides the main conclusions and summarizes them against the objectives set for the study.

## CHAPTER 2

### MATHEMATICAL MODELING

In this chapter, the mathematical formulation of acoustic wave propagation in ducts with single, double, and triple expansion chambers using transfer matrix method are discussed.

Assumptions used to derive the basic equations of the acoustic models are listed as follow:

- Studied 2-D geometry.
- The fluid inside the duct and expansion chambers is air.
- Air flow in the expansion chamber is neglected.
- There is negligible heat transfer through the expansion chamber silencer.
- There is no perforated material inside the expansion chamber.
- Expansion chambers walls not absorptive.
- No wave reflection occurs at downstream end of silencer. This means anechoic termination is considered.

## 2.1 ACOUSTICS WAVE IN DUCTS

The governing equation which describe sound waves of harmonic nature in time in a duct is[18]

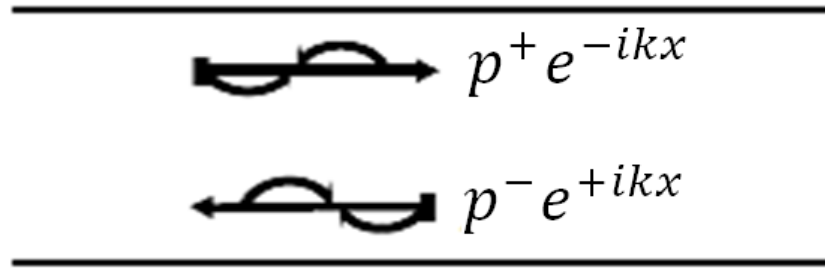
$$\nabla^2 p + k^2 p = 0 \quad , k = \frac{\omega}{c} \quad (2.1)$$

Where  $k$  is the wave number,  $\omega$  is the angular velocity,  $c$  is the speed of the sound, and  $p$  is the acoustics pressure.

The solution of the wave equation in the  $x$ -direction is written in the form[18]

$$p(x) = p^+ e^{-ikx} + p^- e^{+ikx} \quad (2.2)$$

Where  $p(x)$  have two plane waves, one with amplitude  $p^+$  moving in the positive  $x$ -direction and the other with amplitude  $p^-$  moving in the negative  $x$ -direction as shown in Figure 2.1.



**Figure 2.1: Plane wave in a duct.**

By using Euler's equation for motion in a medium, the volume velocity is determined as follow[18]:

$$U(x) = \frac{AP(x)}{\rho_0 c} \quad (2.3)$$



Where  $\rho_0$  is the air density, and  $A$  is the cross sectional area of the duct.

## 2.2 TRANSMISSION LOSS

There are three measures used to assess the acoustic performance of mufflers or silencers. These include the noise reduction (NR), the insertion loss (IL), and the transmission loss (TL). The transmission loss (TL) will be used throughout this thesis. It is the difference in the sound pressure level between the incident waves entering and the transmitted waves exiting the silencer. Transmission loss (TL) is independent of the source and requires an anechoic termination at the downstream end.

The theoretical transmission loss is defined as the logarithmic ratio of incident to transmitted acoustic pressure amplitude into an anechoic termination as Equation 2.4 [39]:

$$TL = 20 \text{ Log}_{10}(\tau) \quad (2.4)$$

Where  $\tau = \frac{T_1}{T_n}$ , and  $T_1$  is the pressure amplitude of the acoustic wave incident expansion chambers and  $T_n$  is the pressure amplitude of the transmitted acoustic wave exiting expansion chambers.

## 2.3 TRANSFER MATRIX METHOD, TMM

The transmission phenomenon in a duct with more than one expansion chamber can be considered by several methods. One of them is the transfer matrix method which provides a suitable framework to obtain theoretical values for transmission loss. Adopting acoustic pressure  $P$  and volume velocity  $U$  as the two state variables[40], every junction in the duct system can be related to the previous junction by a matrix. Then, a global transfer matrix

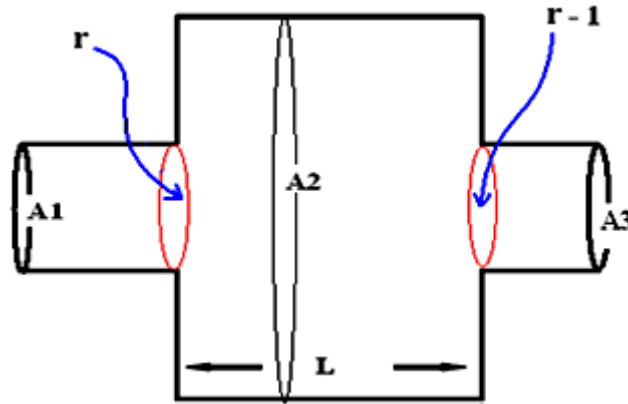
can relate variables at the final junction to those at the first junction by applying continuity conditions of pressure and volume velocity as follows:

$$\begin{bmatrix} P(r) \\ U(r) \end{bmatrix} = \begin{bmatrix} \text{Transfer matrix for the } r_{th} \text{ junction (2 X 2)} \end{bmatrix} \begin{bmatrix} P(r-1) \\ U(r-1) \end{bmatrix} \quad (2.5)$$

Where  $[P(r) \ U(r)]^T$  are called the state vector at the upstream point  $r$ ,

$[P(r-1) \ U(r-1)]^T$  are referred to the state vector at the downstream point

$r-1$ . See Figure 2.2.



**Figure 2.2: General representation of single expansion chamber with upstream and downstream position.**

The transfer matrix for the  $r_{th}$  element can be denoted by  $[T_r]$  [17, 35] as

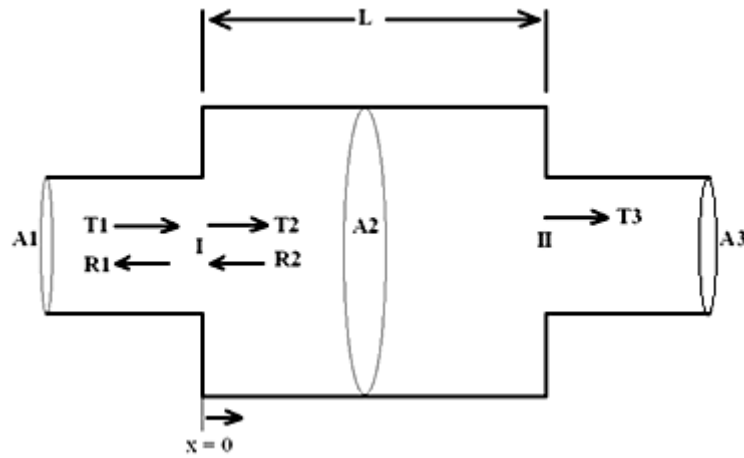
$$[T_r] = [T_1][T_2] \dots [T_{n-1}][T_n] \quad (2.6)$$

An advantage of using the transfer matrix method, to describe silencer effectiveness, is that one can connect more than one expansion chambers by multiplying the transfer matrices of all elements expansion chambers.

## 2.4 TL OF SINGLEEXPANSION CHAMBERSILENCER USINGTMM

### 2.4.1 Transfer Matrix at Junction I

The case of a duct with singleexpansion chamberis shown in Figure 2.3. Transfer matrix method used to find the transfer matrix equation at junction I as follows:



**Figure2.3:Schematic view of single expansion chamber silencer where  $A_j$  :cross sectional areas,  $L_j$  :lengths.**

The continuity pressure equation at the junction between the duct and expansion chamber becomes as following:

$$P_{T1} + P_{R1} = P_{T2} + P_{R2} \quad (2.7)$$

Equation 2.2, one can obtain the following:

$$T_1 e^{i(\omega t - kx)} + R_1 e^{i(\omega t + kx)} = T_2 e^{i(\omega t - kx)} + R_2 e^{i(\omega t + kx)} \quad (2.8)$$

Let junction I be the origin of the frame of reference, then x equals zero at junction I

$$T_1 + R_1 = T_2 + R_2 \quad (2.9)$$

The continuity of volume velocity equation is

$$U_{T1} + U_{R1} = U_{T2} + U_{R2} \quad (2.10)$$

Using Equation 2.3

$$\frac{A_1 P_{T1}}{\rho_0 c} - \frac{A_1 P_{R1}}{\rho_0 c} = \frac{A_2 P_{T2}}{\rho_0 c} - \frac{A_2 P_{R2}}{\rho_0 c} \quad (2.11)$$

This gives,

$$A_1(P_{T1} - P_{R1}) = A_2(P_{T2} - P_{R2}) \quad (2.12)$$

The minus sign come from the direction of the velocity. After using Equation 2.2

$$A_1(T_1 e^{i(\omega t - kx)} - R_1 e^{i(\omega t + kx)}) = A_2(T_2 e^{i(\omega t - kx)} - R_2 e^{i(\omega t + kx)}) \quad (2.13)$$

At junction I, Equation 2.13 becomes,

$$T_1 - R_1 = m (T_2 - R_2) \quad (2.14)$$

Where  $m = \frac{A_2}{A_1}$ , area ratio.

Now, the first transfer matrix at junction I can be obtained from Equations 2.9 and 2.14 as

$$\begin{bmatrix} T_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1+m}{2} & \frac{1-m}{2} \\ \frac{1-m}{2} & \frac{1+m}{2} \end{bmatrix} \begin{bmatrix} T_2 \\ R_2 \end{bmatrix} \quad (2.15)$$

### 2.4.2 Transfer Matrix at Junction II

At junction II,  $x = L$  (expansion chamber length), then the continuity conditions become

$$T_2 e^{-ikL} + R_2 e^{ikL} = T_3 \quad (2.16)$$

$$m(T_2 e^{-ikL} - R_2 e^{ikL}) = T_3 \quad (2.17)$$

So the second transfer matrix at junction II obtained from Equations 2.16 and 2.17as

$$\begin{bmatrix} T_2 \\ R_2 \end{bmatrix} = \begin{bmatrix} \frac{m+1}{2m} e^{ikL} & \frac{m-1}{2m} e^{ikL} \\ \frac{m-1}{2m} e^{-ikL} & \frac{m+1}{2m} e^{-ikL} \end{bmatrix} \begin{bmatrix} T_3 \\ R_3 \end{bmatrix} \quad (2.18)$$

The zero value in final vector comes from the no reflection condition downstream where the duct has an anechoic termination.

### 2.4.3 Transmission Loss of Single Expansion chamber Silencer

The whole transfer matrix of single expansion chamber is obtained by multiplying the first and the second transfer matrices of expansion chamber junctions. Substituting Equations 2.15 into 2.18 to get the whole transfer matrix, as Equation 2.19,

$$\begin{bmatrix} T_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1+m}{2} & \frac{1-m}{2} \\ \frac{1-m}{2} & \frac{1+m}{2} \end{bmatrix} \begin{bmatrix} \frac{m+1}{2m} e^{ikL} & \frac{m-1}{2m} e^{ikL} \\ \frac{m-1}{2m} e^{-ikL} & \frac{m+1}{2m} e^{-ikL} \end{bmatrix} \begin{bmatrix} T_3 \\ 0 \end{bmatrix} \quad (2.19)$$

This equation gives the relationship between the pressure amplitude of the acoustic wave incident and the pressure amplitude of the transmitted acoustic wave exiting the expansion chamber.

The transmission loss is obtained by using Equations 2.4 and 2.19:

$$TL = 20 \log_{10} \left( \frac{(m+1)^2}{4m} e^{ikL} + \frac{(1-m)(m-1)}{4m} e^{-ikL} \right) \quad (2.20)$$

To find the expansion chamber lengths that give maximum and minimum transmission loss. So, convert the exponential function to cosine and sine functions as

$$TL = 20 \log_{10}(\cos kL + \alpha \sin kL) \quad (2.21)$$

$$\text{Where } \alpha = \frac{m^2 + 1}{2m}$$

The extremum points of the logarithm term inside Equation 2.21 present the maximum and minimum of transmission loss of Equation 2.21. Mathematically, to find the extremum points of complex equation it is required to take the derivative of the modulus of that equation then equals it with 0. So, the modulus of the logarithm term inside Equation 2.21 as follows:

$$\mu = \sqrt{(\cos kL)^2 + (\alpha \sin kL)^2} \quad (2.22)$$

$$\frac{\partial (\mu)^2}{\partial L} = -2k \cos kL \sin kL + 2k\alpha^2 \cos kL \sin kL \quad (2.23)$$

$$0 = k(-1 + \alpha^2) \sin 2kL \quad (2.24)$$

$\alpha \neq 0$  at any value of  $m$ , where  $m$  should be greater than one.  $k = 0$ , which means  $f = 0$ .  $\sin 2kL = 0$  which means the lengths that give maximum and minimum transmission loss are

$$L = q \frac{330}{4f} \quad (2.25)$$

Where  $q = 0, 2, 4, \dots$  etc give minimum points and  $q = 1, 3, 5, \dots$  etc give maximum points (second derivative used to distinguish between maximum and minimum points).

Now, to find the area ratios those give maximum and minimum transmission loss, one obtained by taking the derivative of Equation 2.22 as

$$\frac{\partial (\mu)^2}{\partial m} = 0 \quad (2.26)$$

This leads us to the following expression

$$0 = \frac{m^4 - 1}{2m^3} \sin kL \quad (2.27)$$

The solution is  $0 = m^4 - 1$  which gives  $m = 1$ , but this solution is not acceptable because the area ratio should be greater than one. That means the transmission loss increases by increasing the area ratio.

## 2.5 SINGLE EXPANSION CHAMBER SILENCER ANALYSIS (NUMERICAL EXAMPLE)

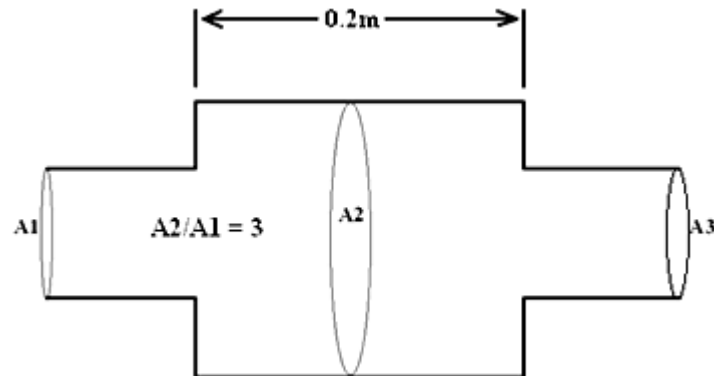
As example, consider simple expansion chamber with 0.2 m length and cross sectional area ratio ( $\frac{A_2}{A_1}$ ) equals 3 as Figure 2.4 shows. To get maximum attenuation, wave reflection must

$$L = n \frac{330}{4f} = n \frac{\lambda}{4}$$

Taking  $n = 1$  
$$L = \frac{\lambda}{4}$$

This gives  $\lambda = 0.8 \text{ m}$   $\Rightarrow$

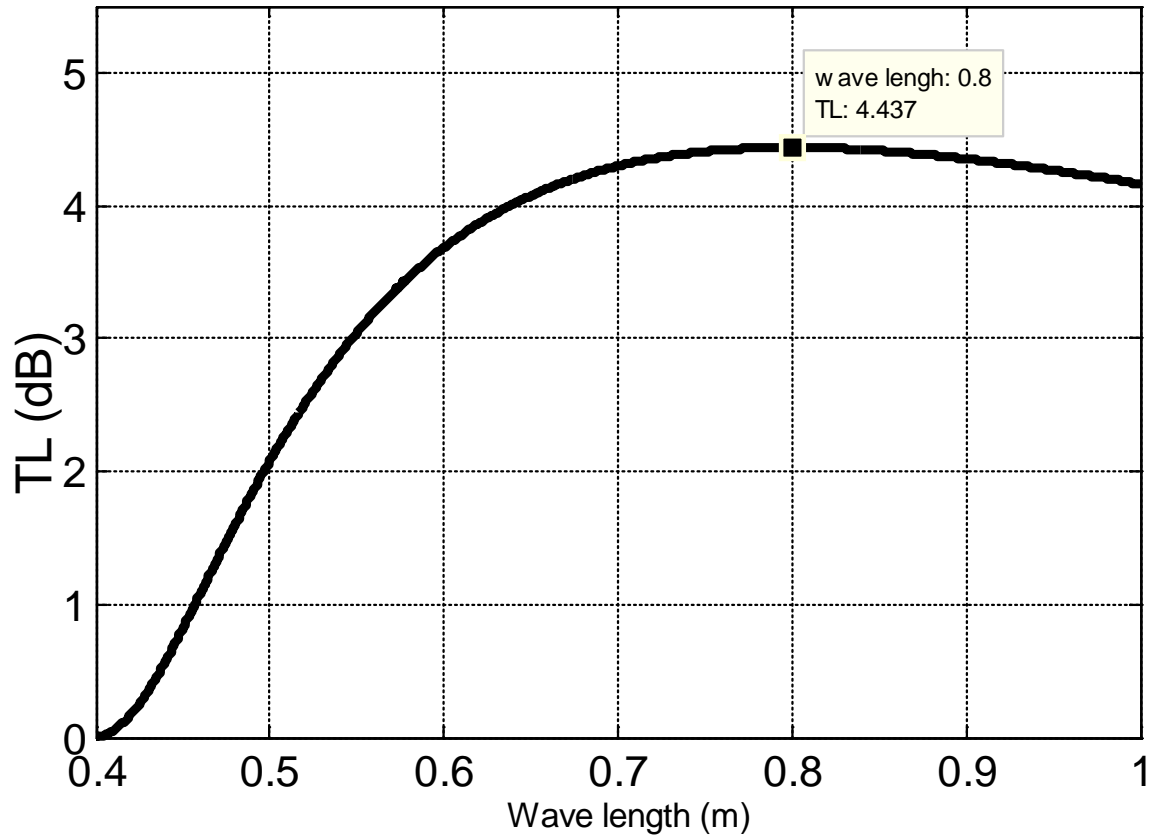
Now, the corresponding frequency  $\Rightarrow f = \frac{c}{\lambda} = \frac{330}{0.8} = 412.5 \text{ Hz}$



**Figure 2.4: An example of single expansion chambers silencer.**

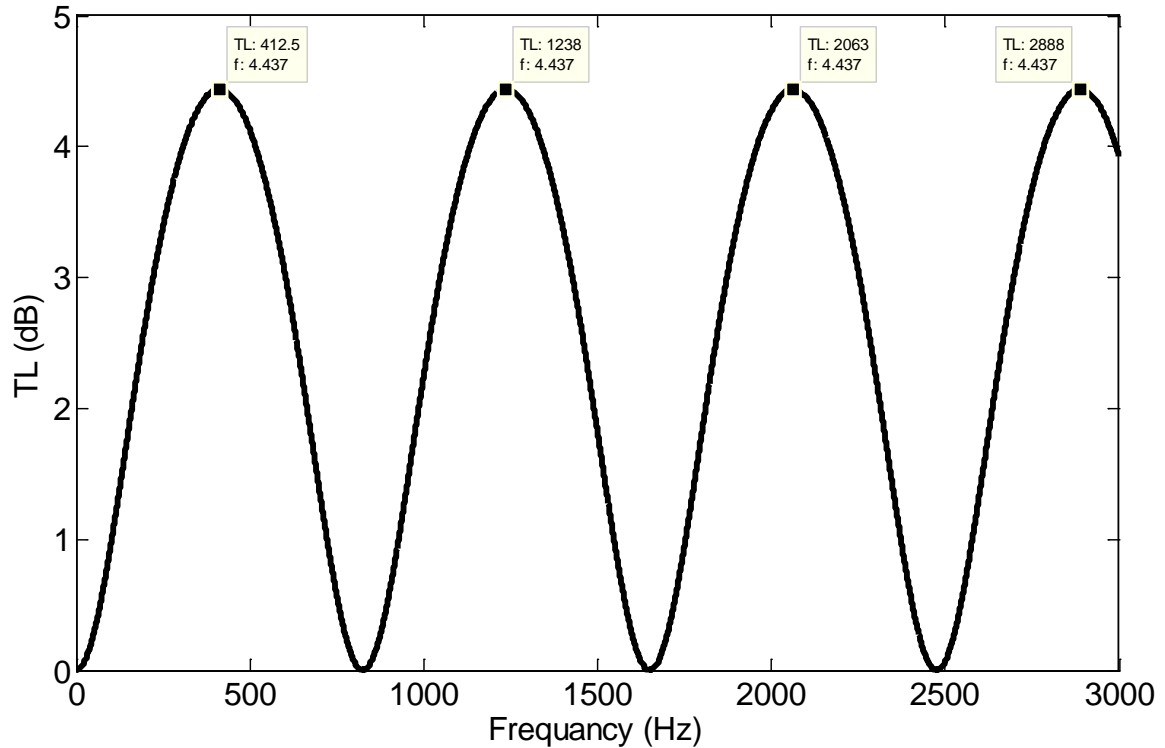
Figure 2.5 shows the transmission loss of the silencer shown in Figure 2.4 as a function of wave length. One can observe that the maximum transmission loss happens at wave length equals 0.8 m which is in agreement with previous analysis. So, one can design the dimensions of the silencer that gives maximum transmission loss at any frequency want to attenuate.





**Figure 2.5: Effect of wave length on the TL of single expansion chambers silencer.**

Typical transmission loss as a function of frequency of single expansion chamber silencer with 0.2 m length ( $L$ ) and area ratio ( $\frac{A_2}{A_1}$ ) equals 3 shown in Figure 2.6. The maximum transmission loss is 4.463 which occur at specific frequencies, like 412.5, 1238, 2063, and 2888 Hz.



**Figure 2.6: Transmission loss curve of single expansion chamber silencer ( $L = 0.2$  m,  $m = 3$ )**

## 2.6 OVERALL TRANSMISSION LOSS

The frequency range of most interest in noise control engineering varies from 20 to 20000 Hz, which is the frequency range hearing by human.

In Figure 2.6, performance property is performed with the use of transmission loss for all frequencies between 0 to 3000 Hz. These results do not explain how much the effectiveness of the silencer for all frequencies because attenuating noise wave depends on silencer parameters which is already optimizing for all frequencies. Overall transmission loss is a numerical value reflecting the effectiveness of the silencers at all frequencies less than 3000 Hz.

**Table 2.1:One – tenth octave band center frequencies.**

Number	Frequency	Number	Frequency	Number	Frequency	Number	Frequency
1	47.5	16	134.5	31	381	46	1078
2	51	17	144.5	32	408.5	47	1155
3	54.5	18	155	33	437.5	48	1238
4	58.5	19	166	34	469	49	1327
5	<b>63</b>	20	178	35	<b>500</b>	50	1422
6	67.5	21	190.5	36	539	51	1524
7	72	22	204	37	577.5	52	1633.5
8	77.5	23	219	38	619	53	1750.5
9	83	24	234.5	39	663.5	54	1876
10	89	25	<b>250</b>	40	711	55	<b>2000</b>
11	95	26	269.5	41	762	56	2155
12	102	27	289	42	816.5	57	2310
13	109.5	28	309.5	43	875	58	2476
14	117	29	331.5	44	938	59	2653.5
15	<b>125</b>	30	335.5	45	<b>1000</b>	60	2844

Table 2.1 presents the center band frequencies for one - tenth octave band which is specified using Equations.

$$\frac{f_{n+1}}{f_n} = 2^{0.1} \quad (2.28)$$

$$f_c = \sqrt{f_n f_{n+1}} \quad (2.29)$$

Where  $f_n$  and  $f_{n+1}$  are lower and upper band limits, respectively and  $f_c$  is the centerband frequency[39]. Equations 2.28 and 2.29 are the basic equations used by the American

National Standards Institute (ANSI) to find the band limits and center frequencies of one, third, tenth octave band frequencies.

To find overall transmission loss for a silencer, firstly, calculate the transmission loss at every center frequency of one-tenth octave band frequencies that is mentioned in Table 2.1 then computes the overall transmission loss (OLT) using

$$OLT = 10 \log \left( \sum_{i=1}^n 10^{[TL_i/10]} \right) \quad (2.30)$$

As an example, the overall transmission loss of the silencer shown in Figure 2.4 is 20.2939 dB at all frequencies less than 3000 Hz.

## 2.7 EFFECT OF CHANGING GEOMETRY

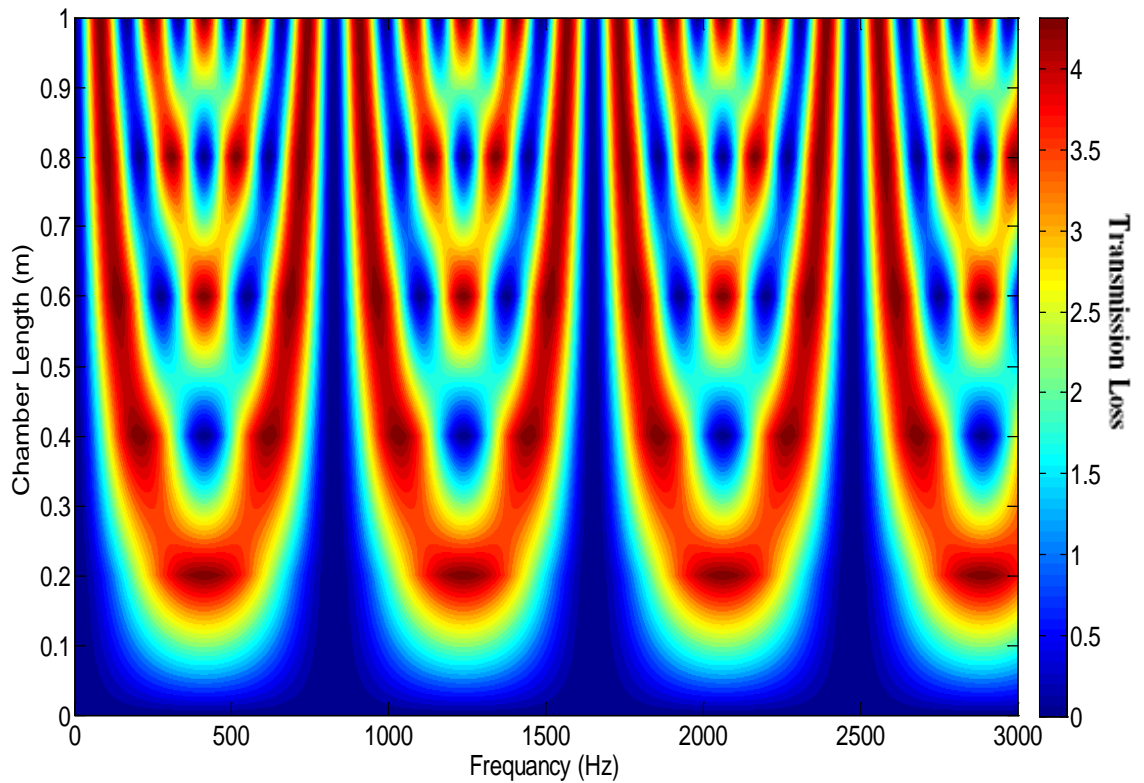
### 2.7.1 Chamber Length

The effect of changing expansion chamber length (L) on transmission loss is visualized as a two dimensional contour plot using Equation 2.20.

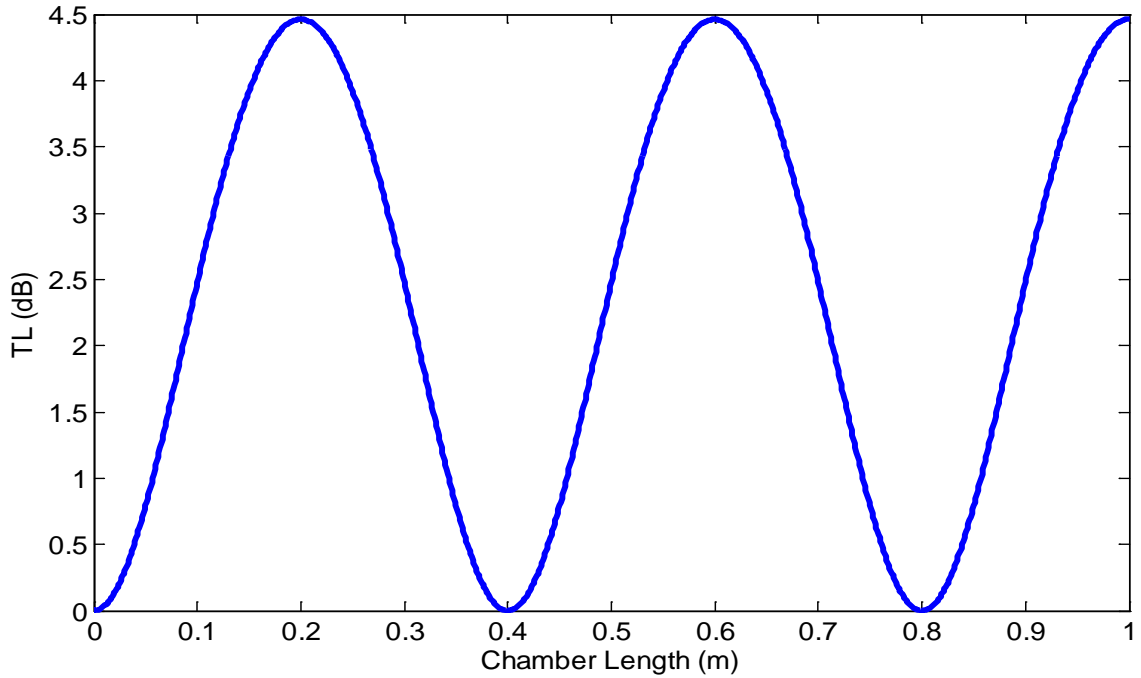
Examining Figure 2.7, one can observe at a specific frequency there are set of expansion chamber lengths which give maximum transmission loss. On the other hand, there are other set of expansion chamber lengths which don't give any transmission loss at all as shown in section 2.5.

Figure 2.8 presents the transmission loss affected by changing expansion chamber length (L) from zero to one meter at the specific frequency 412.5 Hz. It indicates that the expansion chamber lengths 0.2, 0.6, and 1 m give maximum transmission loss which is in

agreement with Equation 2.25. The transmission loss goes to zero when the expansion chamber length is 0.4 or 0.8 m which is in agreement with Equation 2.25. Figure 2.8 indicate how much the chamber length that gives maximum transmission loss at any frequency want to attenuate.



**Figure 2.7: Transmission loss contour of single expansion chamber silencer by changing the expansion chamber length (L), area ratio = 3.**



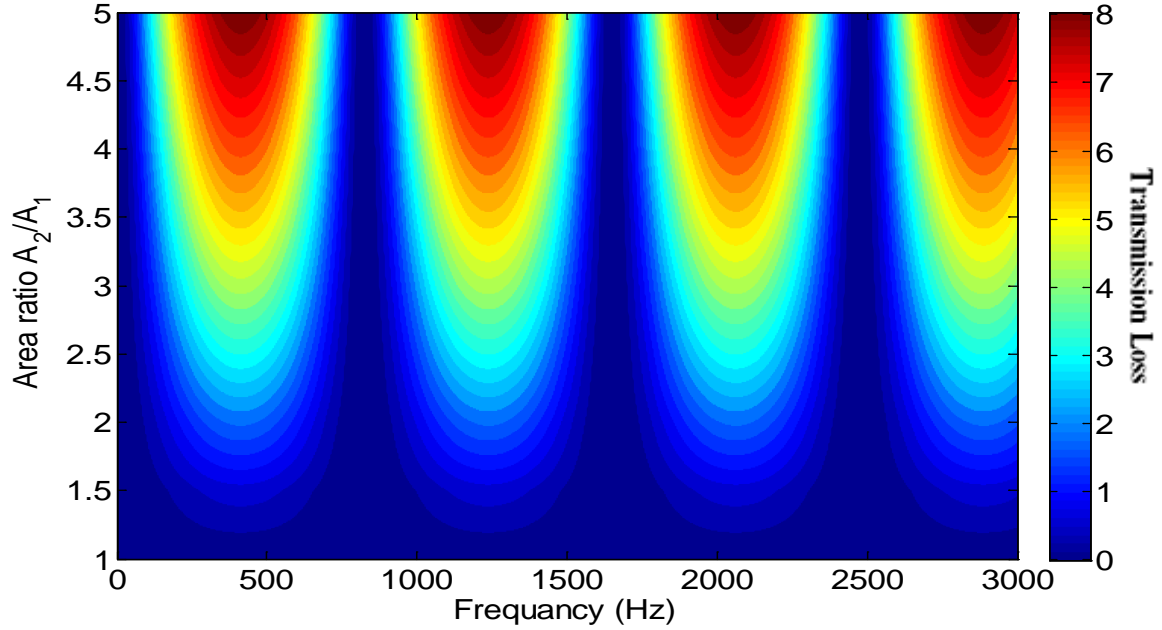
**Figure 2.8: Transmission loss of single expansion chamber silencer by changing expansion chamber length (L) ( $f = 412.5$  Hz, area ratio = 3).**

### 2.7.2 Area Ratio

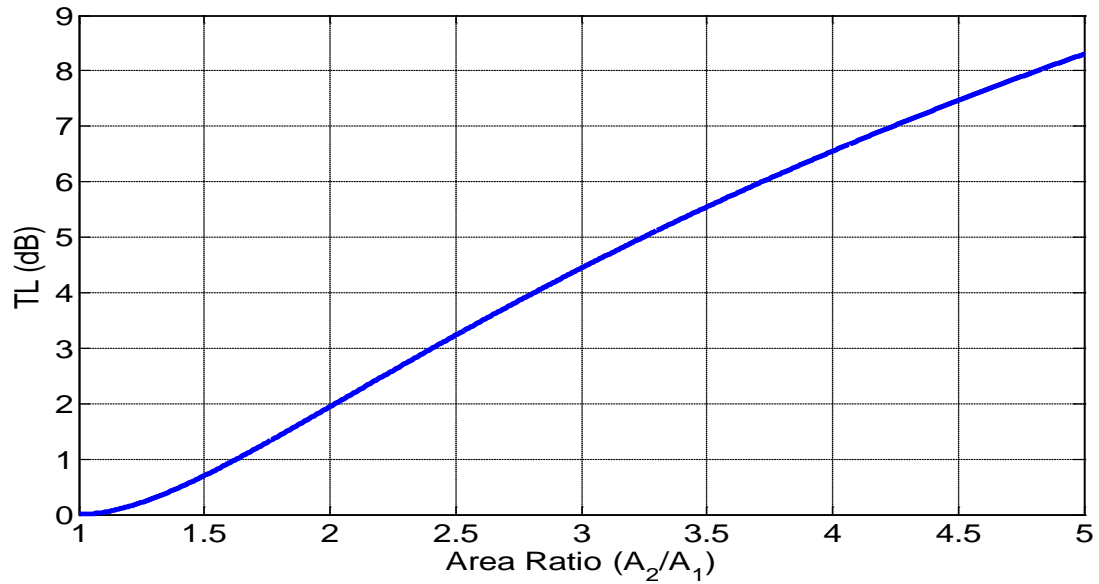
Equation 2.20 used to produce Figure 2.9 which is a two dimensional contour plot showing the transmission loss as a function of frequency and area ratio ( $\frac{A_2}{A_1}$ ). The transmission loss increases when area ratio increases. Hence, the maximum transmission loss occurs at certain frequencies (412.5, 1238, 2063, and 2888 Hz) depends on Equation 2.25. For example, at the frequency 412.5 Hz the maximum transmission loss happen when area ratio ( $\frac{A_2}{A_1}$ ) equals five.

Figure 2.10 shows cross section of Figure 2.9 at frequency 412.5 Hz which represents the transmission loss of the single expansion chamber silencer as a function of area

ratio ( $\frac{A_2}{A_1}$ ). We can conclude that the transmission loss increases continuously by increasing the area ratio ( $\frac{A_2}{A_1}$ ).



**Figure 2.9:** Transmission loss contour of single expansion chamber silencer by changing the area ratio ( $A_2/A_1$ ),  $L = 0.2$  m.



**Figure 2.10:** Transmission loss of single expansion chamber silencer by changing the expansion chamber Area ratio ( $f = 412.5$  Hz,  $L = 0.2$  m.).

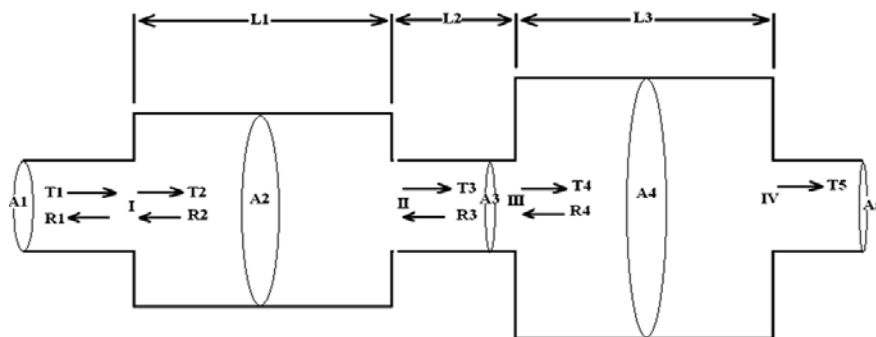
## CHAPTER 3

# ANALYSIS OF SILENCERS HAVING A MULTIPLE EXPANSION CHAMBERS

In this chapter the analysis of acoustic attenuation due to a double and a triple expansion chamber is shown. Parametric study is carried out on the change of transmission loss as a function of investigated the area ratio and the length of the expansion chambers.

### 3.1 DOUBLE EXPANSION CHAMBER ANALYSIS

Figure 3.1 shows the geometry of double expansion chamber reactive silencer. Silencer geometry has two expansion chambers with equal or unequal areas and lengths. The double expansion chamber silencer is an element commonly used for reducing the acoustical noise.



**Figure 3.1: Schematic of double expansion chamber silencer  $A_j$  : cross sectional areas,  $L_j$  : lengths.**



### 3.1.1 TL OF DOUBLE EXPANSION CHAMBERSILENCER USING TMM

The transfer matrix of double expansion chamber silencer is obtained in a similar way to the single expansion chamber case. It takes the following form.

$$\begin{bmatrix} T_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1+m}{2} & \frac{1-m}{2} \\ \frac{1-m}{2} & \frac{1+m}{2} \end{bmatrix} \begin{bmatrix} \frac{m+1}{2m} e^{ikL_1} & \frac{m-1}{2m} e^{ikL_1} \\ \frac{m-1}{2m} e^{-ikL_1} & \frac{m+1}{2m} e^{-ikL_1} \end{bmatrix} \begin{bmatrix} \frac{1+n}{2} e^{ikL_2} & \frac{1-n}{2} e^{ikL_2} \\ \frac{1-n}{2} e^{-ikL_2} & \frac{1+n}{2} e^{-ikL_2} \end{bmatrix} \cdots \quad (2.31)$$

$$\begin{bmatrix} \frac{n+1}{2n} e^{ikL_3} & \frac{n-1}{2n} e^{ikL_3} \\ \frac{n-1}{2n} e^{-ikL_3} & \frac{n+1}{2n} e^{-ikL_3} \end{bmatrix} \begin{bmatrix} T_5 \\ 0 \end{bmatrix}$$

$$\text{Where } m = \frac{A_2}{A_1}, n = \frac{A_4}{A_3}$$

Equation 3.2 presents the transmission loss obtained using Equations 2.4 and 3.1:

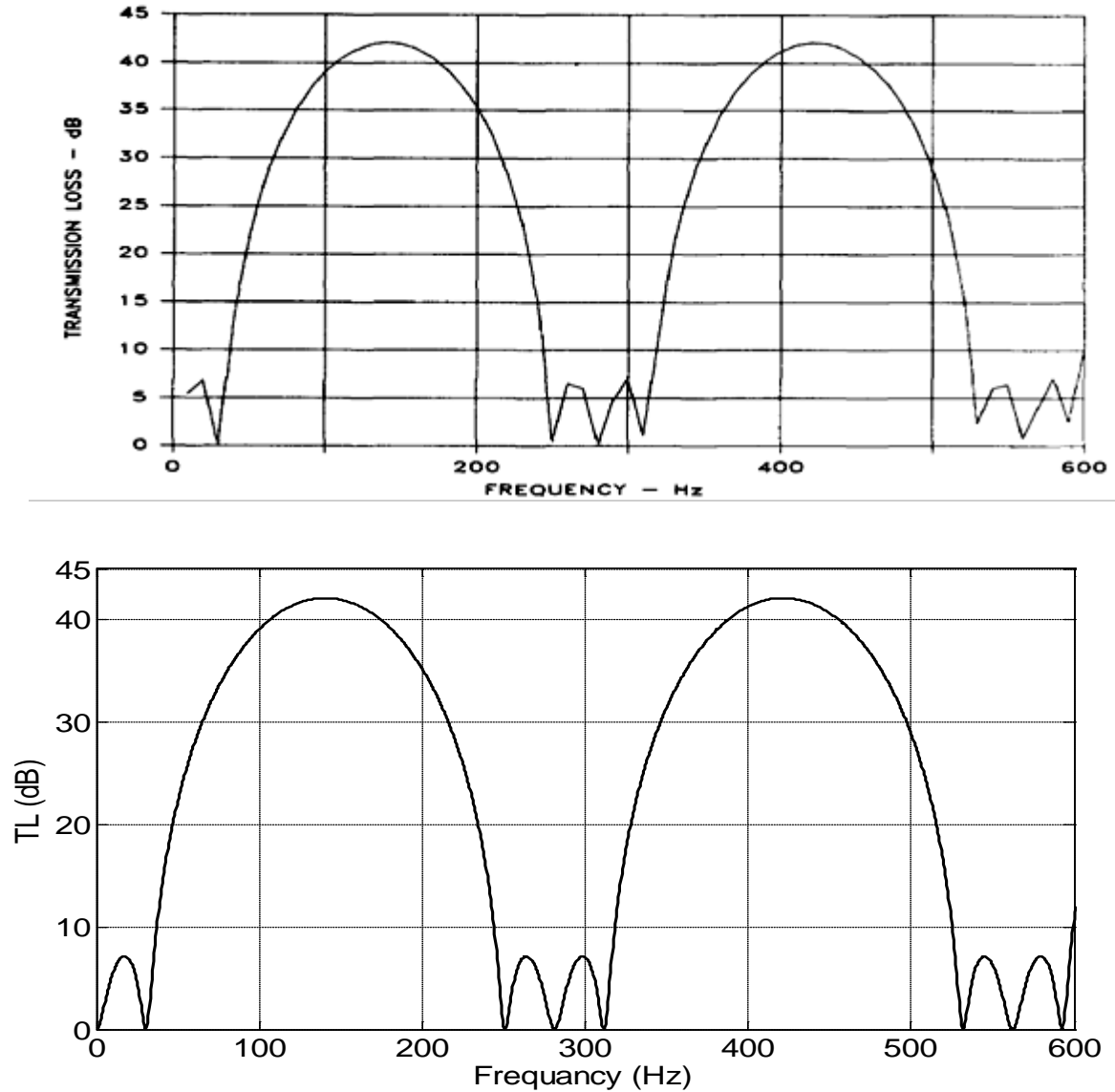
$$TL =$$

$$20 \log_{10} \left[ \left( \frac{(m+1)^2}{4m} e^{ikL_1} + \frac{(1-m)(m-1)}{4m} e^{-ikL_1} \right) \left( \frac{(1+n)^2}{4n} e^{ik(L_2+L_3)} + \frac{(1-n)(n-1)}{4n} e^{ik(L_2-L_3)} \right) + \right. \\ \left. \left( \frac{(1+m)(m-1)}{4m} e^{ikL_1} + \frac{(1-m)(m+1)}{4m} e^{-ikL_1} \right) \left( \frac{(1-n)(n+1)}{4n} e^{ik(L_3-L_2)} + \frac{(1+n)(n-1)}{4n} e^{-ik(L_2+L_3)} \right) \right] \quad (3.2)$$

### 3.1.2 Mathematical Model Validation

The mathematical model has been validated by calculated the transmission loss of double expansion chamber silencer with same geometrical parameter that used by Lamancusa[7] as shown in Figure 3.2. Figure 3.2 (a) produced by Lamancusa which presents the transmission loss of double expansion chamber silencer. Figure 3.2 (b) shows the same

transmission loss results that got by Lamancusa. This means, the mathematical model is valid.

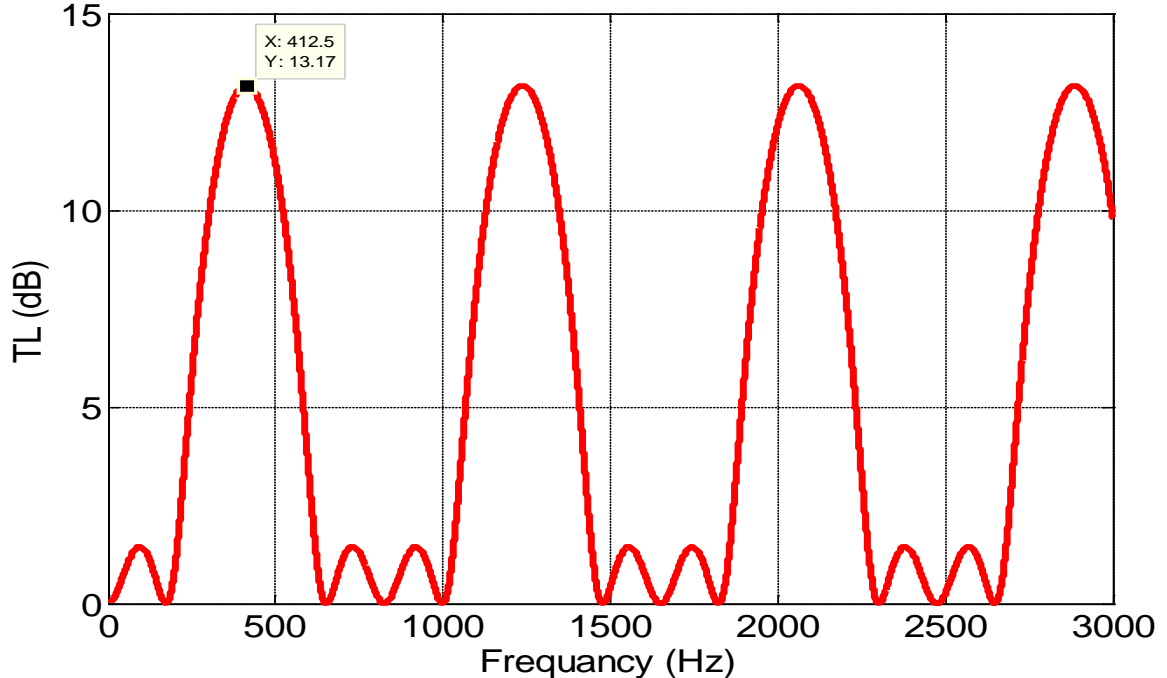


**Figure 3.2 Transmission Loss of double expansion chamber silencer with equally sized Chambers of 0.61 m and 0.61m connecting tubes, area ratio = 16.**

**(a) Lamancusa results (b) validation results**

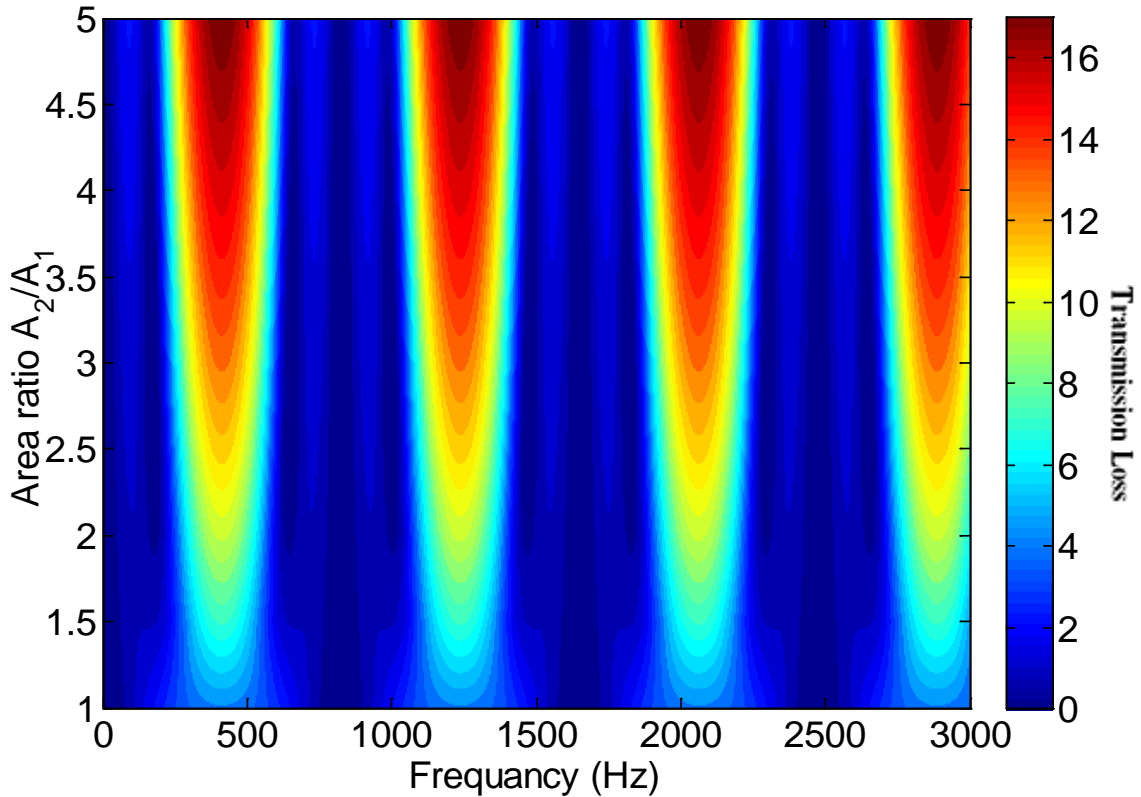
### 3.1.3 Effect of changing geometry

Figure 3.3 shows the typical transmission loss of double expansion chamber silencer with 0.2 m lengths ( $L_1, L_2, L_3$ ) and area ratios ( $m, n$ ) equal 3. The maximum transmission loss is 13.17 dB which occurs at the frequencies 412.5, 1238, 2063, and 2888 Hz. Comparing Figure 3.3 with 2.5, one can note a difference between single and double expansion chambers transmission loss spectra. Also, it is noted that major wave reflection regions are separated by double small ripples. The overall transmission loss of the double expansion chamber silencer is 24.9258 dB. Thus, increasing the number of expansion chambers in the silencer gives higher overall transmission loss as Figures 2.5 and 3.3 showed. Figure 3.3 agrees well with results behavior given for the same problem by Lamancusa[7].



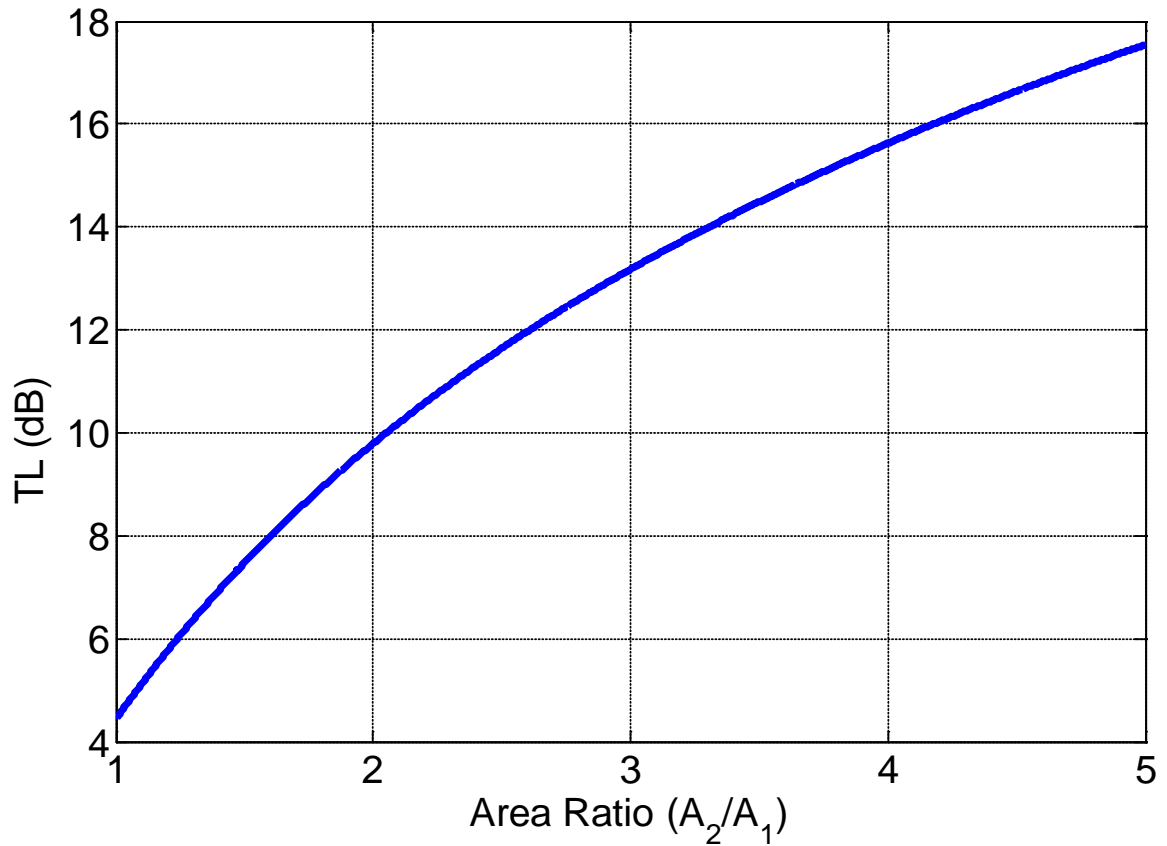
**Figure 3.3: Transmission loss curve for double expansion chamber (lengths = 0.2m, area ratios = 3).**

The next contour figures have been generated using Equation 3.2. Figure 3.4 shows a two dimensional contour plot of transmission loss as a function of frequency and first area ratio ( $\frac{A_2}{A_1}$ ). The transmission loss increases when area ratio increases. The maximum transmission loss occurs at specific frequencies (412.5, 1238, 2063, and 2888 Hz). When the first area ratio ( $\frac{A_2}{A_1}$ ) equals one the maximum transmission loss becomes 4.463 dB. This attenuation comes from the second expansion chamber that has an area ratio ( $\frac{A_4}{A_3}$ ) equals 3 same as transmission loss of single expansion chamber silencer.



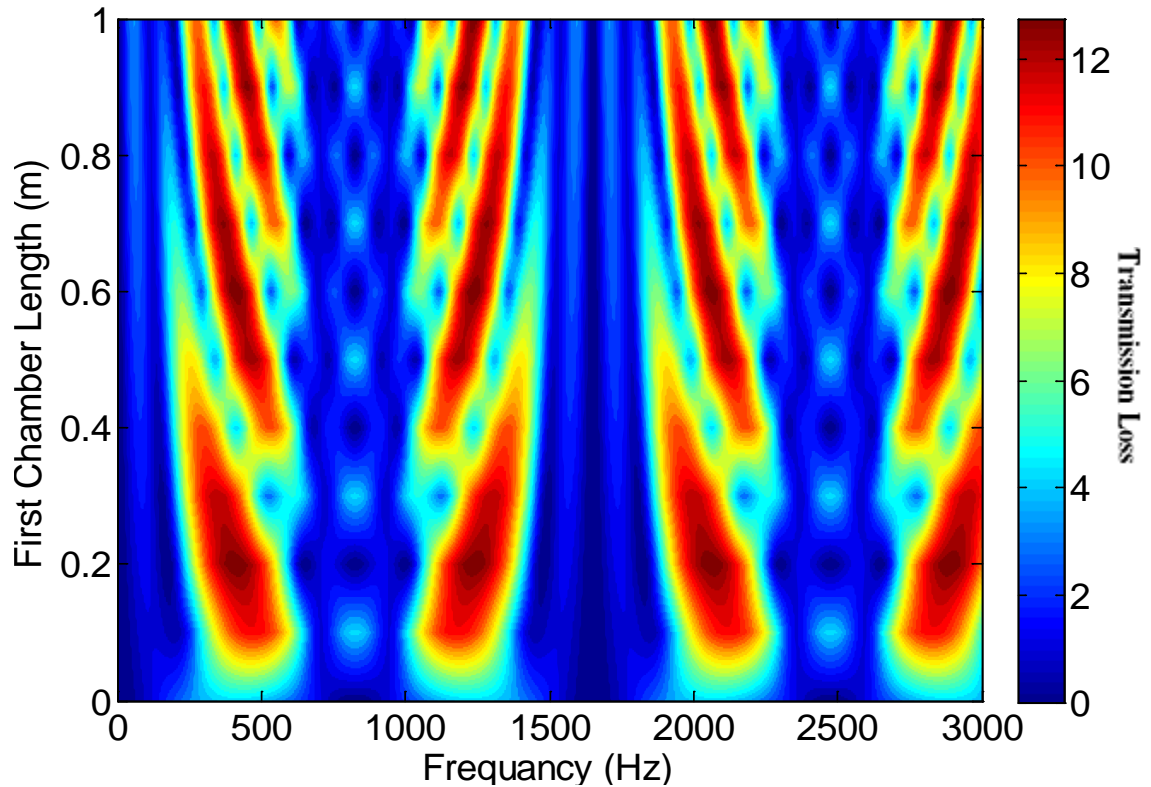
**Figure 3.4: Transmission loss contour of double expansion chamber silencer by changing the area ratio ( $A_2/A_1$ ),  $L_j = 0.2$ .**

Figure 3.5 shows an example of Figure 3.4. It indicates that the transmission loss increases when the first area ratio ( $\frac{A_2}{A_1}$ ) increases. Also, same result is obtained by increasing second area ratio ( $\frac{A_4}{A_3}$ ). The value of transmission loss is calculated to be 4.463 dB when the first expansion chamber area ratio is one and the second area ratio ( $\frac{A_4}{A_3}$ ) is 3. This means that the double expansion chamber silencer performs in a similar manner with a single expansion chamber silencer and it gives same transmission loss result as indicated in Figure 2.5.



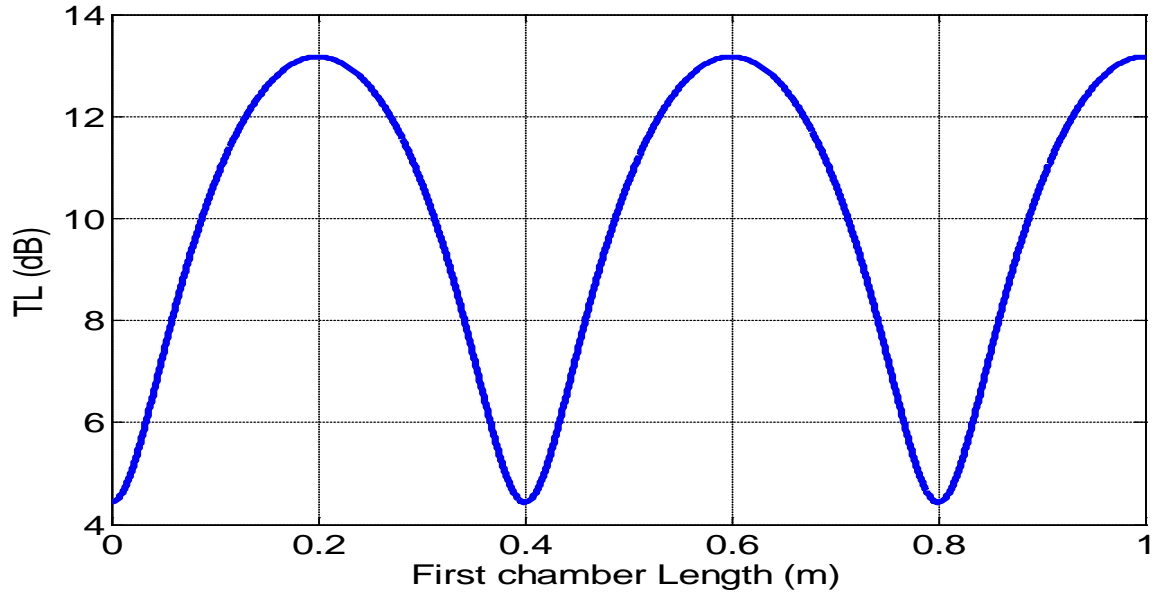
**Figure 3.5: Double expansion chamber silencer transmission loss by changing the first expansion chamber Area ratio ( $f = 412.5$  HZ,  $L_j = 0.2$ ).**

Figure 3.6 shows transmission loss contour of double expansion chamber silencer as a function of frequency and first expansion chamber length ( $L_1$ ). As example, if the expansion chamber length equals 0.2 m, the maximum transmission loss is around 13.17 dB at the frequency 412.5 Hz which is higher than the transmission loss of single expansion chamber silencer.



**Figure 3.6: Transmission loss contour of double expansion chamber silencer by changing the first expansion chamber length ( $L_1$ ), (area ratios = 3).**

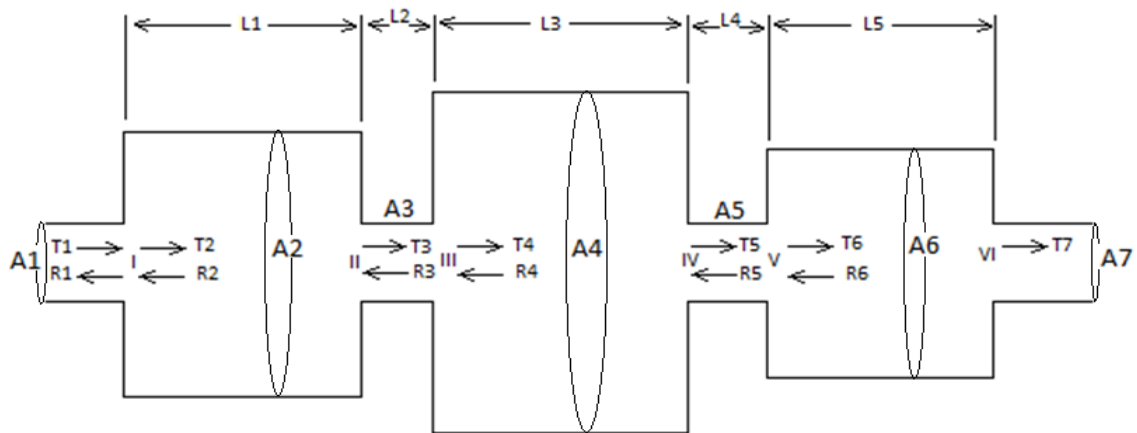
Figure 3.7 indicates that the first expansion chamber lengths ( $L_1$ ) 0.2, 0.6, 1 m have the maximum transmission loss. And similar behavior of Figure 3.6 resulted when studying the transmission loss by changes second expansion chamber length ( $L_3$ ) or the connecting tube length ( $L_2$ ).



**Figure 3.7: Double expansion chamber silencer transmission loss by changing the first expansion chamber length ( $L$ ), ( $f = 412.5$  HZ,  $m$  and  $n = 3$ ).**

### 3.2 TRIPLE EXPANSION CHAMBERSILENCER ANALYSIS

Figure 3.8 shows a triple expansion chamber reactive silencer which is an element used to increase the noise attenuation through the ducts systems.



**Figure 3.8: Schematic of triple expansion chamber silencer where  $A_j$  : cross sectional areas,  $L_j$  : lengths.**

### 3.2.1 TL OF TRIPLE EXPANSION CHAMBERSILENCER USING TMM

The transfer matrix of triple expansion chambersilencer is obtained in a similar manner to double expansion chambersilencer. So, Equation 3.3 presents the whole transfer matrix of the silencer.

$$\begin{bmatrix} T_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1+m}{2} & \frac{1-m}{2} \\ \frac{1-m}{2} & \frac{1+m}{2} \end{bmatrix} \begin{bmatrix} \frac{m+1}{2m} e^{ikL_1} & \frac{m-1}{2m} e^{ikL_1} \\ \frac{m-1}{2m} e^{-ikL_1} & \frac{m+1}{2m} e^{-ikL_1} \end{bmatrix} \begin{bmatrix} \frac{1+n}{2} e^{ikL_2} & \frac{1-n}{2} e^{ikL_2} \\ \frac{1-n}{2} e^{-ikL_2} & \frac{1+n}{2} e^{-ikL_2} \end{bmatrix} \begin{bmatrix} \frac{n+1}{2n} e^{ikL_3} & \frac{n-1}{2n} e^{ikL_3} \\ \frac{n-1}{2n} e^{-ikL_3} & \frac{n+1}{2n} e^{-ikL_3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1+g}{2} e^{ikL_4} & \frac{1-g}{2} e^{ikL_4} \\ \frac{1-g}{2} e^{-ikL_4} & \frac{1+g}{2} e^{-ikL_4} \end{bmatrix} \begin{bmatrix} \frac{g+1}{2g} e^{ikL_5} \\ \frac{g-1}{2g} e^{-ikL_5} \end{bmatrix} \begin{bmatrix} T_7 \\ 0 \end{bmatrix} \quad (3.3)$$

Where  $m = \frac{A_2}{A_1}$ ,  $n = \frac{A_4}{A_3}$ ,  $g = \frac{A_6}{A_5}$

Finally, Equation 3.4 presents the transmission loss of the silencer that obtained using Equations 2.4 and 3.3 as follows:

$$\begin{aligned} TL = 20 \log_{10} & \left( \left( \frac{(m+1)^2}{4m} e^{ikL_1} + \frac{(1-m)(m-1)}{4m} e^{-ikL_1} \right) \left( \frac{(1+n)^2}{4n} e^{ik(L_2+L_3)} + \right. \right. \\ & \left. \frac{(1-n)(n-1)}{4n} e^{ik(L_2-L_3)} \right) \left( \frac{(1+g)^2}{4g} e^{ik(L_4+L_5)} + \frac{(1-g)(g-1)}{4g} e^{ik(L_4-L_5)} \right) + \left( \frac{(1+n)(n-1)}{4n} e^{ik(L_2+L_3)} \right. \\ & \left. \left. + \frac{(1-n)(n+1)}{4n} e^{ik(L_2-L_3)} \right) \left( \frac{(1-g)(g+1)}{4g} e^{ik(L_5-L_4)} + \frac{(1+g)(g-1)}{4g} e^{-ik(L_4+L_5)} \right) \right) + \end{aligned}$$

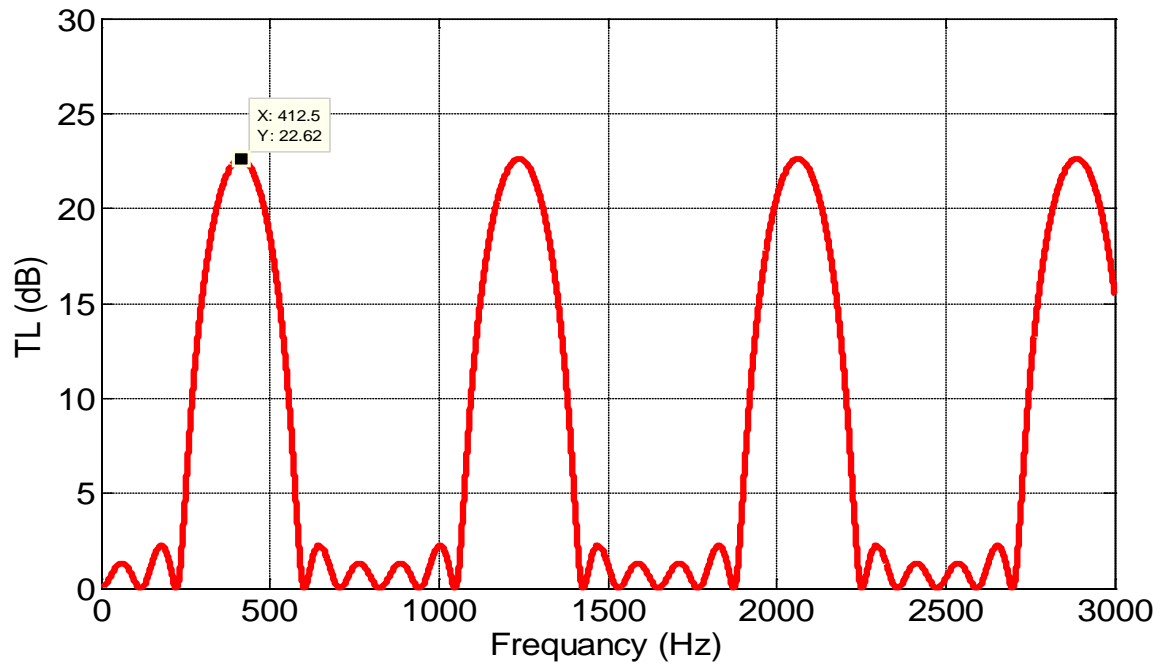


$$\begin{aligned}
& \left( \frac{(1+m)(m-1)}{4m} e^{ikL_1} \right. \\
& \quad \left. + \frac{(1-m)(m+1)}{4m} e^{-ikL_1} \right) \left[ \left( \frac{(1-n)(n+1)}{4n} e^{ik(L_3-L_2)} \right. \right. \\
& \quad \left. \left. + \frac{(1+n)(n-1)}{4n} e^{-ik(L_2+L_3)} \right) \left( \frac{(1+g)^2}{4g} e^{ik(L_4+L_5)} \right. \right. \\
& \quad \left. \left. + \frac{(1-g)(g-1)}{4g} e^{ik(L_4-L_5)} \right) \right. \\
& \quad \left. + \left( \frac{(1-n)(n-1)}{4n} e^{ik(L_3-L_2)} + \frac{(1+n)^2}{4n} e^{-ik(L_2+L_3)} \right) \right. \\
& \quad \left. \left( \frac{(1-g)(g+1)}{4g} e^{ik(L_5-L_4)} + \frac{(1+g)(g-1)}{4g} e^{-ik(L_4+L_5)} \right) \right] \quad (3.4)
\end{aligned}$$

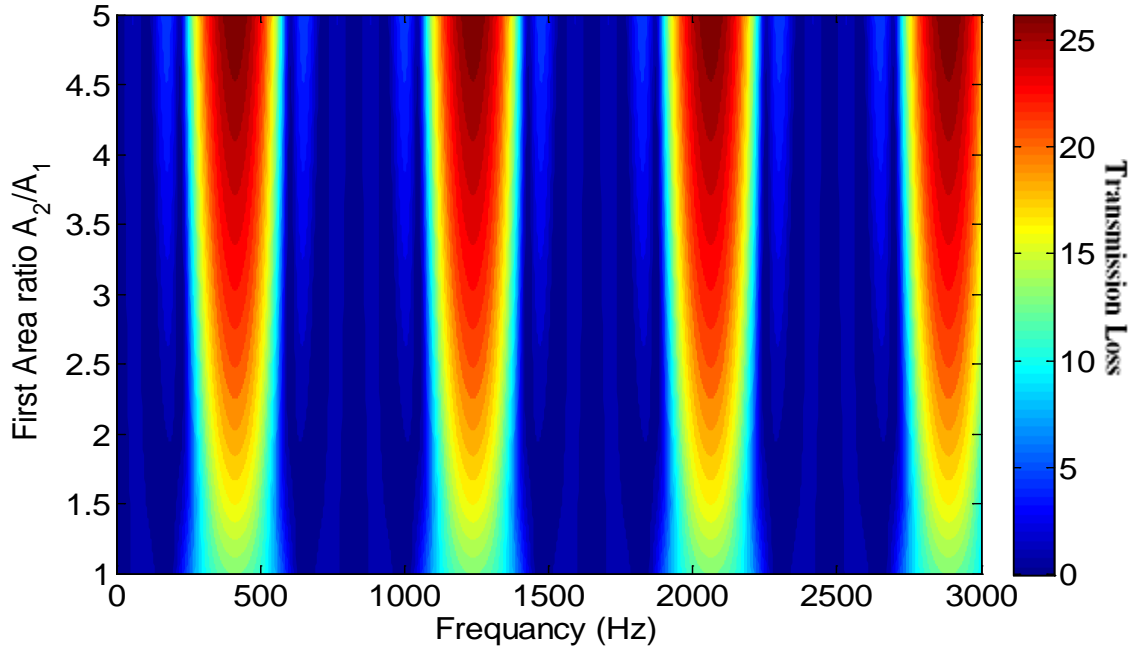
### 3.2.2 Effect of changing geometry

The next figures have been generated using Equation 3.4. Figure 3.9 shows the transmission loss of triple expansion chamber silencer when the expansion chamber and connecting tube lengths ( $L_j$ ) equal 0.2 m and all area ratios ( $m$ ,  $n$ , and  $g$ ) equal 3. The attenuation noise is increased by using triple expansion chamber silencer because the number of expansion chambers is increased. 22.63 dB is the maximum transmission loss which occurs at the frequencies 412.5, 1238, 2063, 2888 Hz. Overall transmission loss is 32.2966 dB. Also, it is noted that major wave reflection regions are separated by four small ripples.

Figure 3.10 shows two dimensional contour plot of the transmission loss as a function of frequency and first area ratio ( $\frac{A_2}{A_1}$ ). The transmission loss increases when the area ratio increases. The maximum transmission loss occurs at the similar frequencies mentioned in double expansion chamber silencer case.

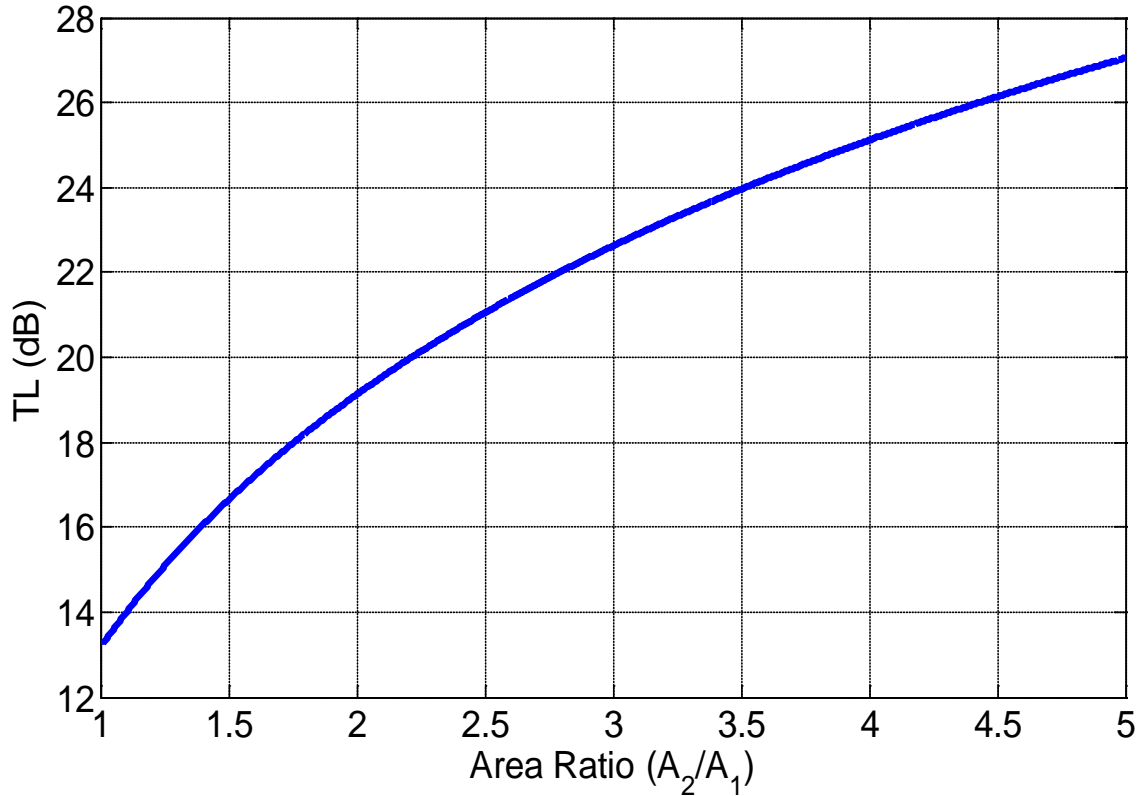


**Figure 3.9: Transmission loss curve of triple expansion chamber silencer ( $L_j = 0.2$  m), (area ratios = 3).**



**Figure 3.10: Transmission loss contour of triple expansion chamber silencer by changing the area ratio ( $A_2/A_1$ ),  $L_j = 0.2$  m.**

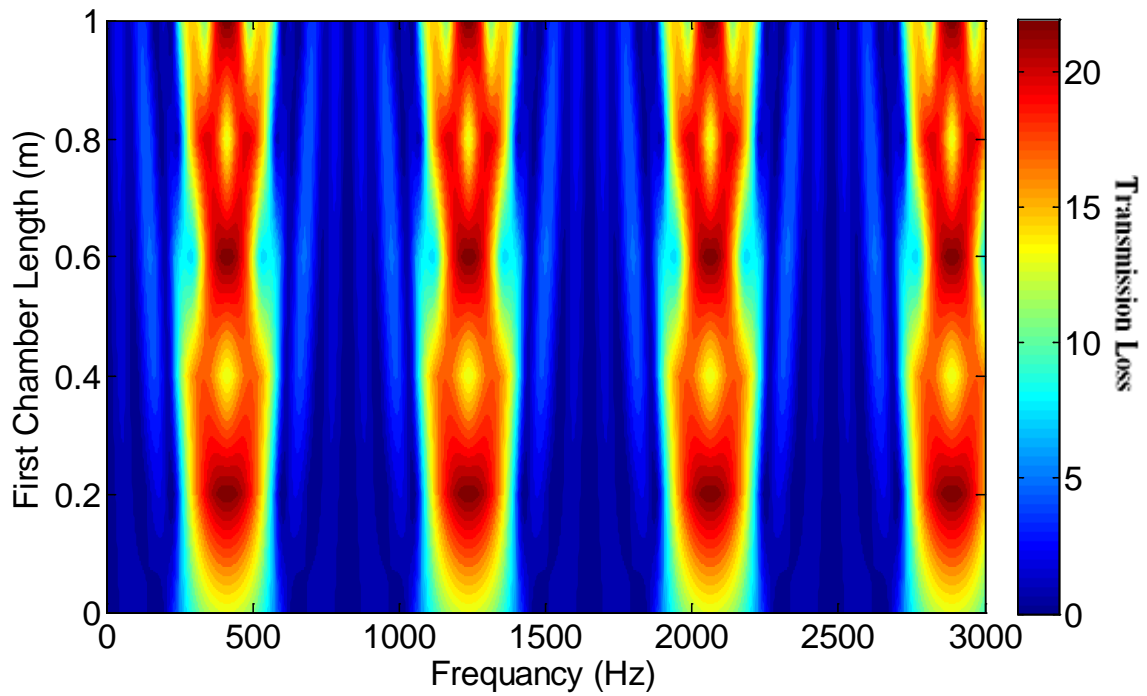
Figure 3.11 shows, the transmission loss increases when first expansion chamber area ratio ( $\frac{A_2}{A_1}$ ) increases. Similar curve obtains when second area ratio ( $\frac{A_4}{A_3}$ ) or third area ratio ( $\frac{A_6}{A_5}$ ) increases. The value of maximum transmission loss is almost 13.7 when first area ratio ( $\frac{A_2}{A_1}$ ) equals one, second and third area ratios ( $\frac{A_4}{A_3} = \frac{A_6}{A_5}$ ) equal three. This means that the triple expansion chamber silencer performs in a similar manner with double expansion chamber silencer and it gives same transmission loss as indicate in Figure 3.3.



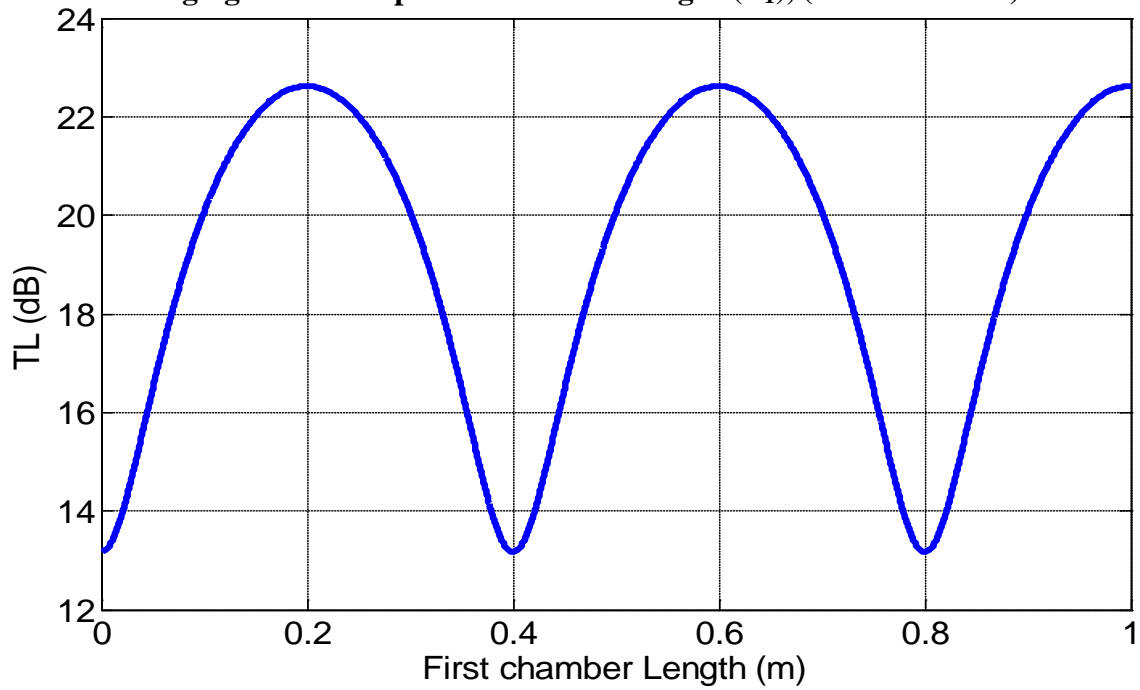
**Figure 3.11: Triple expansion chamber silencer transmission loss by changing the first expansion chamber Area ratio ( $f = 412.5$  HZ),  $L_j = 0.2$  m.**

Figure 3.12 shows the transmission loss contour as a function of frequency and first expansion chamber length  $L_1$ . As example if the expansion chamber length equals 0.6 m, the maximum transmission loss is around 22.62 dB at the frequency equals 412.5 Hz which is more than the maximum transmission loss value of double expansion chamber silencer.

Figure 3.13 has similar behavior of Figure 3.7 with different values of transmission loss due to increase the number expansion chambers. Also this indicates the first expansion chamber lengths 0.2, 0.6 and 1m have maximum transmission loss. And similar curves resulted by change second, third expansion chamber, or connecting tubes lengths ( $L_j$ ).



**Figure 3.12:** Transmission loss contour of triple expansion chamber silencer by changing the first expansion chamber length ( $L_1$ ), (area ratios = 3).

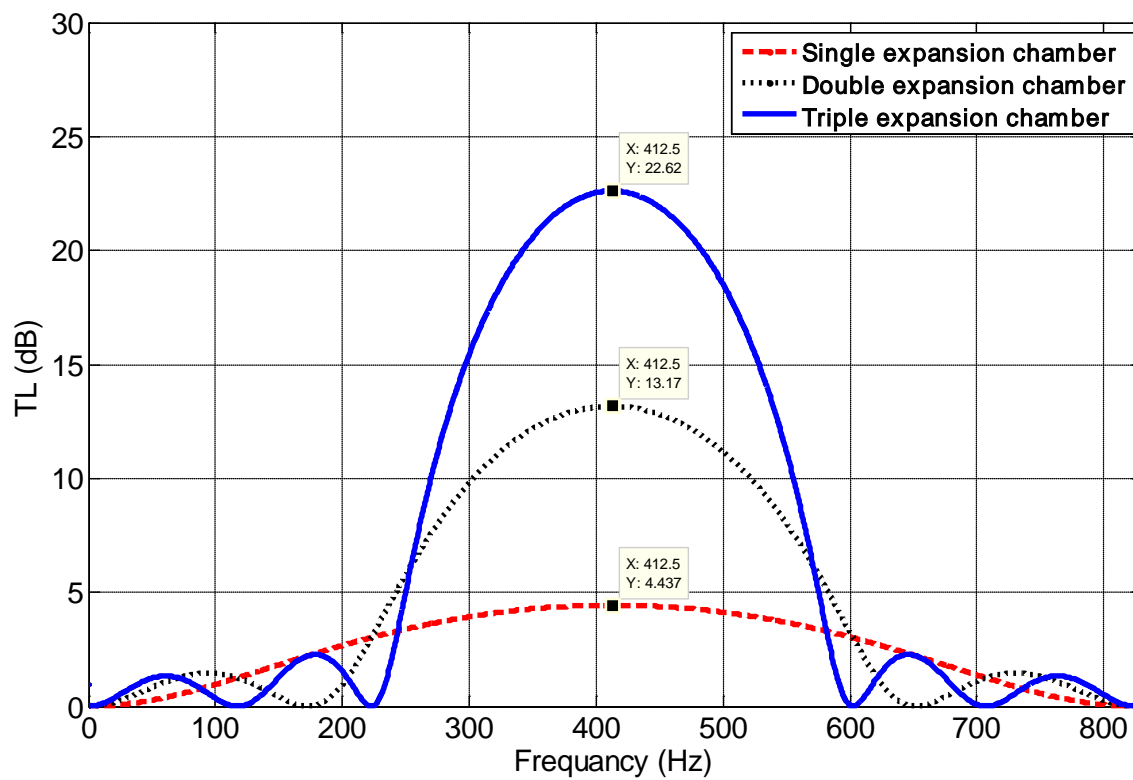


**Figure 3.13:** Triple expansion chamber silencer transmission loss by changing the first expansion chamber length ( $L_1$ ), ( $m, n, g = 3$ ).

### 3.3 COMPARISON BETWEEN THE THREE SILENCER CONFIGURATIONS

In Figure 3.14, a comparison made between three configurations silencers using expansion chambers lengths equal 0.2 m, connecting tubes lengths equal 0.2 m and area ratios (m, n, g) equal 3, within range frequency between zero and 830 Hz.

The best result is given by the triple expansion chamber silencer because it has highest transmission loss value. The value of transmission loss increases linearly with increases the number of expansion chambers used in the silencer. Therefore, the triple expansion chamber gives 22.62 dB transmission loss at the frequency 412.5 Hz, which is the maximum attenuation between three configurations. Hence the transmission loss curve has 380 Hz stopband width which is smallest width in Figure 3.14. The three curves show almost similar difference between the values of maximum transmission loss which is around 9 dB. This means the maximum transmission loss increases by 9 dB when expansion chamber is added to the silencer structure.



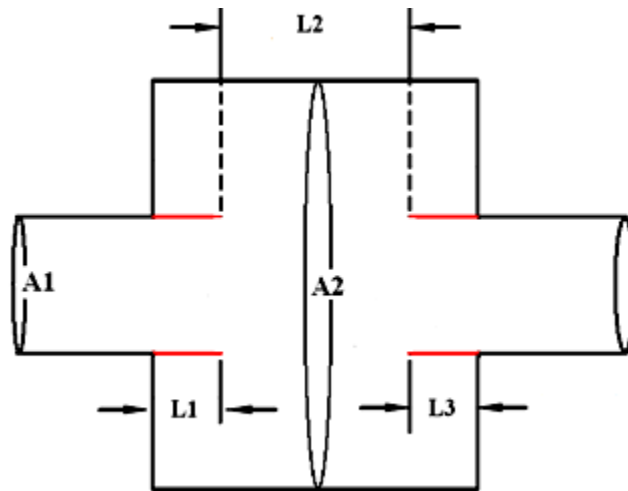
**Figure 3.14: Transmission loss of three configurations silencers.**

## CHAPTER 4

### ENHANCING ATTENUATION (I)

#### INTRODUCING EXTENDED TUBES

In this chapter, mathematical formulation of expansion chamber silencer with extended tube is discussed using transfer matrix method. A schematic of single expansion chamber silencer with extended tube is shown in Figure 4.1. Also, investigate the difference between expansion chamber silencer with and without extended tubes.

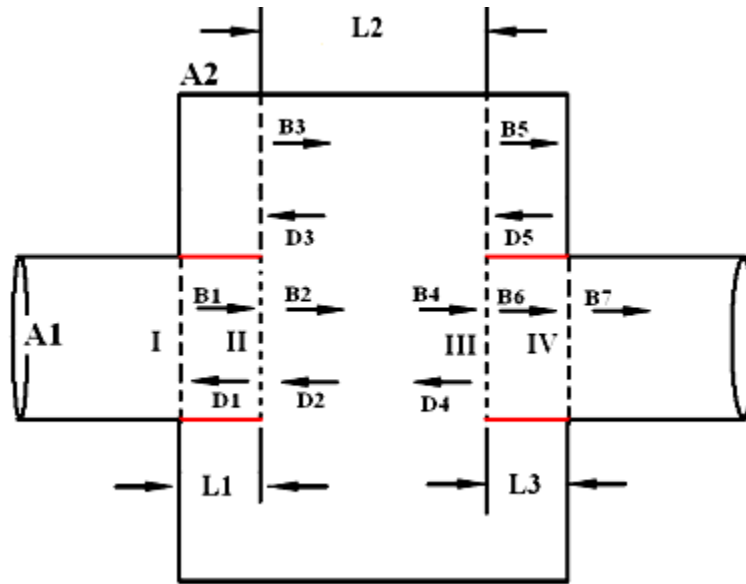


**Figure 4.1:** Schematic view of an expansion chamber silencer with extended tube where  $A_j$  : areas,  $L_j$  : lengths.



In order to obtain the transmission loss using transfer matrix method, as in chapter three, continuity of pressure and continuity of volume velocity equations are applied at each junction in expansion chamber.

For the case of single expansion chamber, let us assume a wave has amplitude  $B_i$  travels forward and a wave with amplitude  $D_i$  travels backward as Figure 4.2 shows.



**Figure 4.2: Single expansion chamber silencer with extended tubes.**

When using expansion chamber silencer with extended tubes Equation 2.5 becomes as follow [40]:

$$\begin{bmatrix} P(r) \\ U(r) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{1,3}} & 1 \end{bmatrix} \begin{bmatrix} P(r-1) \\ U(r-1) \end{bmatrix} \quad (4.1)$$

Where  $Z$  is the impedance at junctions II, III as

$$Z_{1,3} = -i \frac{\rho_o c}{(A_2 - A_{1,3})} \cot(kL_{1,3}) \quad (4.2)$$

## 4.1 MATHEMATICAL MODEL OF SINGLE EXPANSION CHAMBER SILENCER WITH EXTENDED TUBE USING TMM

### 4.1.1 Transfer Matrix at Junctions I and II

Continuity of pressure equation is

$$P_{B1} + P_{D1} = P_{B2} + P_{D2} \quad (4.3)$$

Equation 2.2, one can obtain the following:

$$B_1 e^{i(\omega t - kx)} + D_1 e^{i(\omega t + kx)} = B_2 e^{i(\omega t - kx)} + D_2 e^{i(\omega t + kx)} \quad (4.4)$$

Using Equation 4.1, Equation 4.4 becomes:

$$B_1 e^{i(\omega t - kx)} + D_1 e^{i(\omega t + kx)} = B_3 e^{i(\omega t - kx)} + D_3 e^{i(\omega t + kx)} \quad (4.5)$$

Let junction I be the origin of the frame of reference, then  $x$  equals  $L_1$  at junction II

$$B_1 e^{-ikL_1} + D_1 e^{ikL_1} = B_3 e^{-ikL_1} + D_3 e^{ikL_1} \quad (4.6)$$

The continuity of volume velocity equation and Equation 4.1 at junction II give,

$$U_{B1} + U_{D1} = \frac{1}{Z_1} (P_{B2} + P_{D2}) + U_{B2} + U_{D2} \quad (4.7)$$

Using Equation 2.3 gives,

$$\frac{A_1 P_{B1}}{\rho_0 c} - \frac{A_1 P_{D1}}{\rho_0 c} = \frac{1}{Z_1} (P_{B2} + P_{D2}) + \frac{A_2 P_{B2}}{\rho_0 c} - \frac{A_2 P_{D2}}{\rho_0 c} \quad (4.8)$$

Also, this gives,

$$B_1 e^{-ikL_1} - D_1 e^{ikL_1} = \mu_1 B_3 e^{-ikL_1} - \mu_2 D_3 e^{ikL_1} \quad (4.9)$$

Where

$$\mu_1 = \frac{\frac{\rho_0 c}{A_1}}{-i \frac{\rho_0 c}{(A_2 - A_1)} \cot(kL_1)} + \frac{A_2}{A_1} \quad (4.10)$$

$$\mu_2 = \frac{\frac{\rho_0 c}{A_1}}{i \frac{\rho_0 c}{(A_2 - A_1)} \cot(kL_1)} + \frac{A_2}{A_1} \quad (4.11)$$

Now, the first transfer matrix at junction II can be obtained from Equations 4.6 and 4.9 as.

$$\begin{bmatrix} B_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} \frac{1 + \mu_1}{2} & \frac{1 - \mu_2}{2} e^{2ikL_1} \\ \frac{1 - \mu_1}{2} e^{2ikL_1} & \frac{1 + \mu_2}{2} \end{bmatrix} \begin{bmatrix} B_3 \\ D_3 \end{bmatrix} \quad (4.12)$$

### 4.1.2 Transfer Matrix at Junctions III and IV

The continuity of pressure equation at junction III as following:

$$P_{B4} + P_{D4} = P_{B6} \quad (4.13)$$

At x equals zero, Equation 4.13 gives the following:

$$B_4 + D_4 = B_6 \quad (4.14)$$

But the wave transfers a distance  $L_2$  from junction II to junction III. So, Equation 4.14 becomes as

$$B_3 e^{-ikL_2} + D_3 e^{ikL_2} = B_6 \quad (4.15)$$

Using continuity of volume velocity equation with Equation 4.1 obtains the following:

$$U_{B4} + U_{D4} = \frac{1}{Z_3} (P_{B6}) + U_{B6} \quad (4.16)$$

Using Equations 2.2 and 4.2, Equation 4.16 becomes as

$$B_3 e^{-ikL_2} - D_3 e^{ikL_2} = \mu_3 B_6 \quad (4.17)$$

Where

$$\mu_3 = \frac{\frac{\rho_1 c}{A_2}}{-i \frac{\rho_0 c}{(A_2 - A_1)} \cot(kL_3)} + \frac{A_1}{A_2} \quad (4.18)$$

So the second transfer matrix at junction III obtained from Equations 4.15 and 4.17 as

$$\begin{bmatrix} B_3 \\ D_3 \end{bmatrix} = \begin{bmatrix} \frac{1 + \mu_3}{2} e^{ikL_2} \\ \frac{1 - \mu_3}{2} e^{-ikL_2} \end{bmatrix} \begin{bmatrix} B_6 \\ 0 \end{bmatrix} \quad (4.19)$$

Finally the wave transfers from junction III to junction IV with a distance  $L_3$  as

$$B_7 = B_6 e^{-ikL_3} \quad (4.20)$$

### 4.1.3 Transmission Loss Equation of Single Expansion chamber Silencer with Extended tubes

The whole transfer matrix of single expansion chamber silencer with extended tubes is obtained by multiplying first and second transfer matrices. So the whole transfer matrix is obtained using Equations 4.12, 4.19 and 4.20 as

$$\begin{bmatrix} B_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} \frac{1+\mu_1}{2} & \frac{1-\mu_2}{2} e^{2ikL_1} \\ \frac{1-\mu_1}{2} e^{2ikL_1} & \frac{1+\mu_2}{2} \end{bmatrix} \begin{bmatrix} \frac{1+\mu_3}{2} e^{ik(L_3+L_2)} \\ \frac{1-\mu_3}{2} e^{ik(L_3-L_2)} \end{bmatrix} \begin{bmatrix} B_7 \\ 0 \end{bmatrix} \quad (4.21)$$

$$TL = 20 \log_{10} \left( \frac{(\mu_1 + 1)(\mu_3 + 1)}{4} e^{ik(L_3+L_2)} + \frac{(1 - \mu_2)(1 - \mu_3)}{4} e^{-ik(2L_1-L_2+L_3)} \right) \quad (4.22)$$

Equation 4.22 gives the relationship between pressure amplitude of incident wave and pressure amplitude of transmitted wave exiting expansion chamber silencer.

## 4.2 TL OF DOUBLE EXPANSION CHAMBERSILENCER WITH EXTENDED TUBE USING TMM

Figure 4.3 presents double expansion chamber silencer with extended tubes. The transfer matrix of double expansion chamber with extended tube is obtained in a similar manner of single expansion chamber silencer with extended tube case.

$$\begin{bmatrix} B_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} \frac{1+\mu_1}{2} & \frac{1-\mu_2}{2} e^{2ikL_{11}} \\ \frac{1-\mu_1}{2} e^{2ikL_{11}} & \frac{1+\mu_2}{2} \end{bmatrix} \begin{bmatrix} \frac{m+1}{2m} e^{ikL_1} & \frac{m-1}{2m} e^{ikL_1} \\ \frac{m-1}{2m} e^{-ikL_1} & \frac{m+1}{2m} e^{-ikL_1} \end{bmatrix} \begin{bmatrix} \frac{1+n}{2} e^{ikL_2} & \frac{1-n}{2} e^{ikL_2} \\ \frac{1-n}{2} e^{-ikL_2} & \frac{1+n}{2} e^{-ikL_2} \end{bmatrix} \cdots \begin{bmatrix} \frac{1+\mu_3}{2} e^{ik(L_3+L_{33})} \\ \frac{1-\mu_3}{2} e^{ik(L_{33}-L_3)} \end{bmatrix} \begin{bmatrix} B_8 \\ 0 \end{bmatrix} \quad (4.23)$$

The transmission loss can be obtained using Equations 2.4 and 4.23.

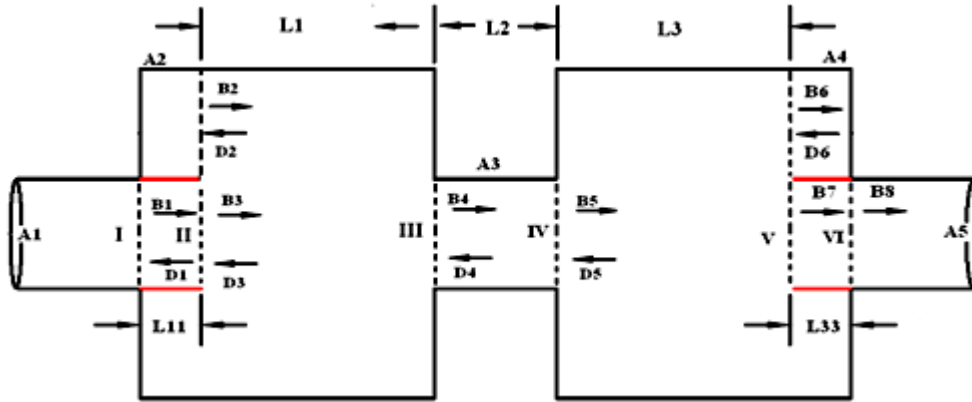


Figure 4.3: Double expansion chambersilencers with extended tubes

### 4.3 TL OF TRIPLE EXPANSION CHAMBER SILENCER WITH EXTENDED TUBES USING TMM

Figure 4.3 presents triple expansion chamber silencer with extended tubes. The transfer matrix of triple expansion chamber silencer gets in a similar way that transfer matrix of double expansion chamber silencer is obtained as

$$\begin{bmatrix} B_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} \frac{1+\mu_1}{2} & \frac{1-\mu_2}{2} e^{2ikL_{11}} \\ \frac{1-\mu_1}{2} e^{2ikL_{11}} & \frac{1+\mu_2}{2} \end{bmatrix} \begin{bmatrix} \frac{m+1}{2m} e^{ikL_1} & \frac{m-1}{2m} e^{ikL_1} \\ \frac{m-1}{2m} e^{-ikL_1} & \frac{m+1}{2m} e^{-ikL_1} \end{bmatrix} \begin{bmatrix} \frac{1+n}{2} e^{ikL_2} & \frac{1-n}{2} e^{ikL_2} \\ \frac{1-n}{2} e^{-ikL_2} & \frac{1+n}{2} e^{-ikL_2} \end{bmatrix} \cdots \\
 \begin{bmatrix} \frac{n+1}{2n} e^{ikL_3} & \frac{n-1}{2n} e^{ikL_3} \\ \frac{n-1}{2n} e^{-ikL_3} & \frac{n+1}{2n} e^{-ikL_3} \end{bmatrix} \begin{bmatrix} \frac{1+g}{2} e^{ikL_4} & \frac{1-g}{2} e^{ikL_4} \\ \frac{1-g}{2} e^{-ikL_4} & \frac{1+g}{2} e^{-ikL_4} \end{bmatrix} \begin{bmatrix} \frac{1+\mu_3}{2} e^{ik(L_{55}+L_5)} \\ \frac{1-\mu_3}{2} e^{ik(L_{55}-L_5)} \end{bmatrix} \begin{bmatrix} B_{10} \\ 0 \end{bmatrix} \quad (4.24)$$

The transmission loss can be obtained using Equations 2.4 and 4.24.

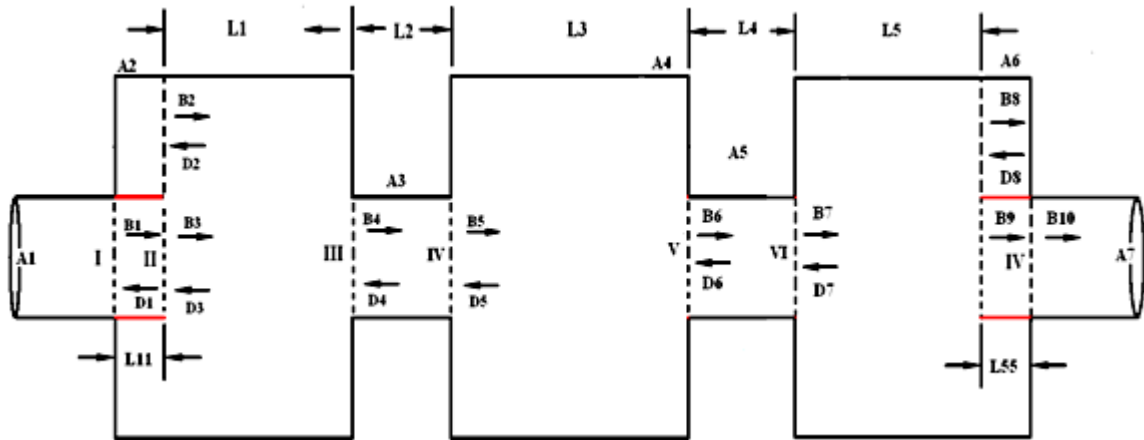


Figure 4.4: Triple expansion chamber silencer with extended tubes

## 4.4 COMPARISON BETWEEN SILENCERS WITH AND WITHOUT EXTENDED TUBES

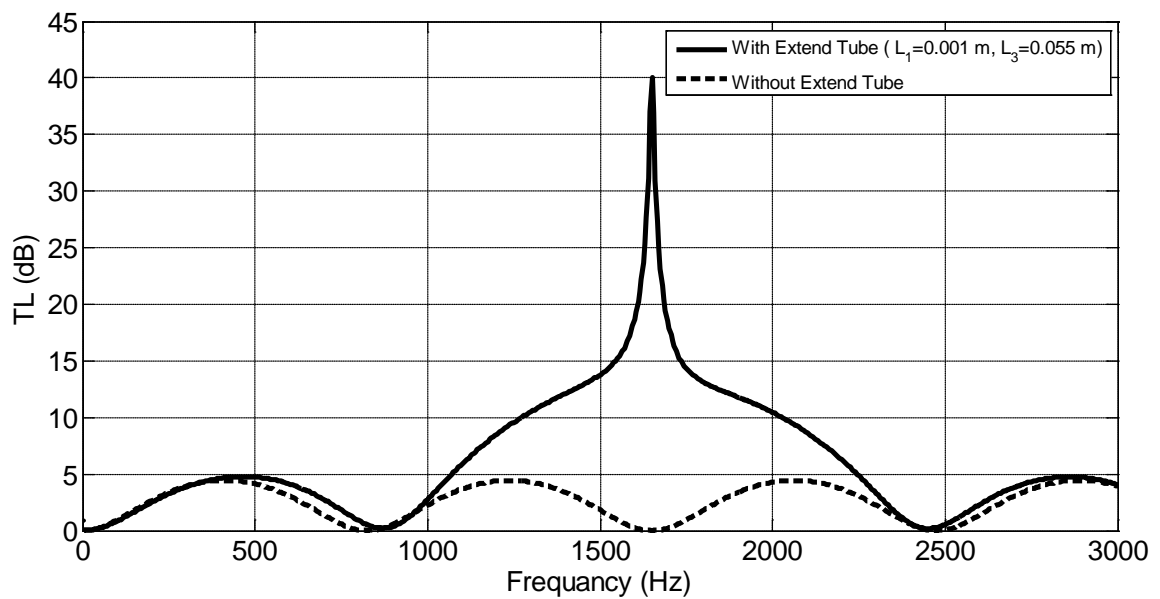
### 4.4.1 Single Expansion chamber Silencer

Table 4.1 shows geometrical parameters of single expansion chamber silencer which is used to calculate the performance property (TL) difference for single expansion chambers silencer with and without extended tubes.

**Table 4.1: Geometrical parameters of single expansion chamber silencer with and without extended tubes.**

Silencer types	Expansion chamber area ( $m^2$ )	Duct area ( $m^2$ )	Expansion chamber length (m)	First extended tube length $L_1$ (m)	Second extended tube length $L_3$ (m)
With extended tubes	0.0942	0.0314	0.2	0.001	0.055
Without extended tubes	0.0942	0.0314	0.2	0	0

Figure 4.5 presents transmission loss difference between single expansion chamber silencers with extended tubes (SET) and without extended tubes (SNET). So, the maximum transmission loss of silencer without extended tubes is 4.437 dB, and it increases to be 40 dB with extended tubes usage. Also, the area under SET curve is larger than the area under SNET curve. This means the silencer with extended tubes is a better choice to attenuate noise wave.



**Figure 4.5: TL of single expansion chamber silencer with and without extended tubes.**

Table 4.2 shows overall transmission loss of single expansion chamber with and without extended tubes. The silencer with extended tubes has overall transmission loss higher than the other one. This means it is better to use extended tubes in single expansion chamber silencer.

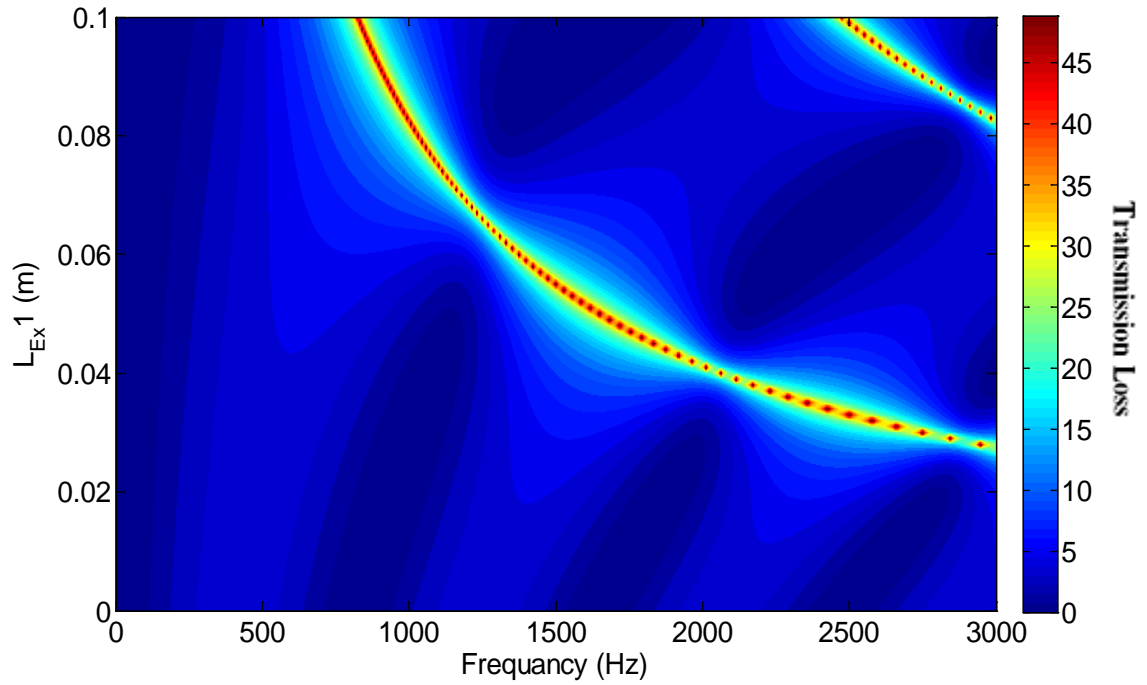


**Table 4.2: OLT of single expansion chamber silencer.**

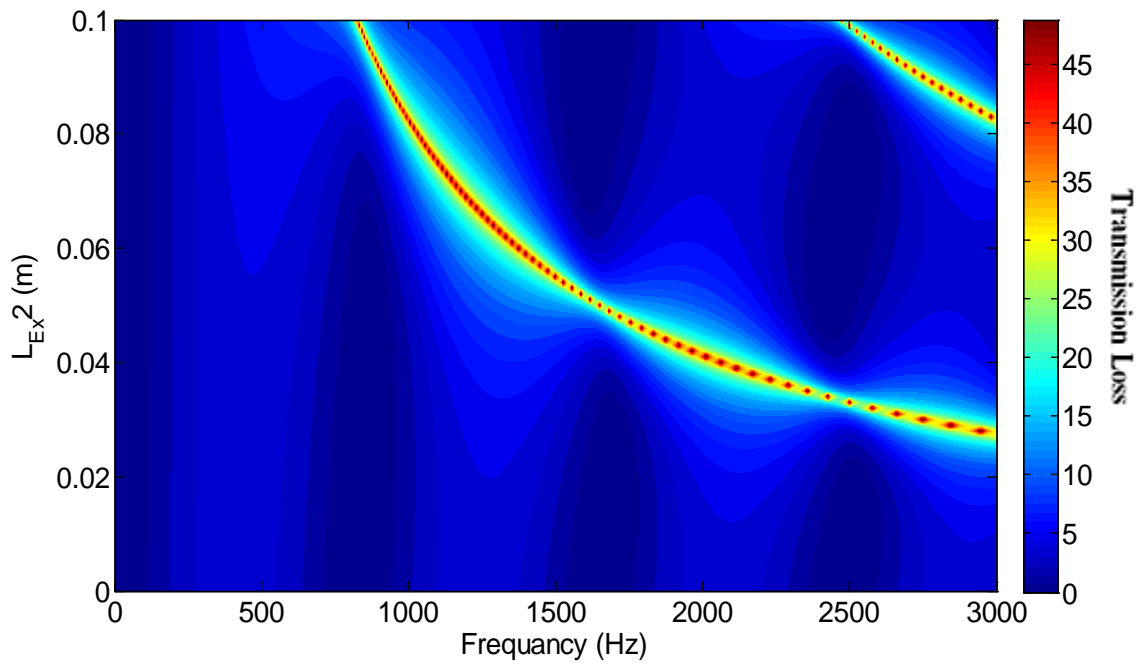
Silencers type depend on extended tubes		OLT (dB)
1	Without extended tubes	20.2939
2	With extended tubes	28.5538

The effect on transmission loss when changing the extended tubes lengths can be shown as two dimensional contour plot using Equation 4.22. Figure 4.6 shows the inlet extended tube length effect on transmission loss. The value of inlet extended tube length, that gives maximum transmission loss, depends on frequency value. Figure 4.7 presents the transmission loss contour affected by the outlet extended tube length.

The extended tubes give more transmission loss at high frequency rather than low frequency. So, from Figures 4.6 and 4.7, one can see the lengths of outlet and inlet extended tubes can be controlled to get high transmission loss at any frequency need to attenuate.



**Figure 4.6: Transmission loss of single expansion chamber silencer with extended tubes by changes inlet extended tube length.**

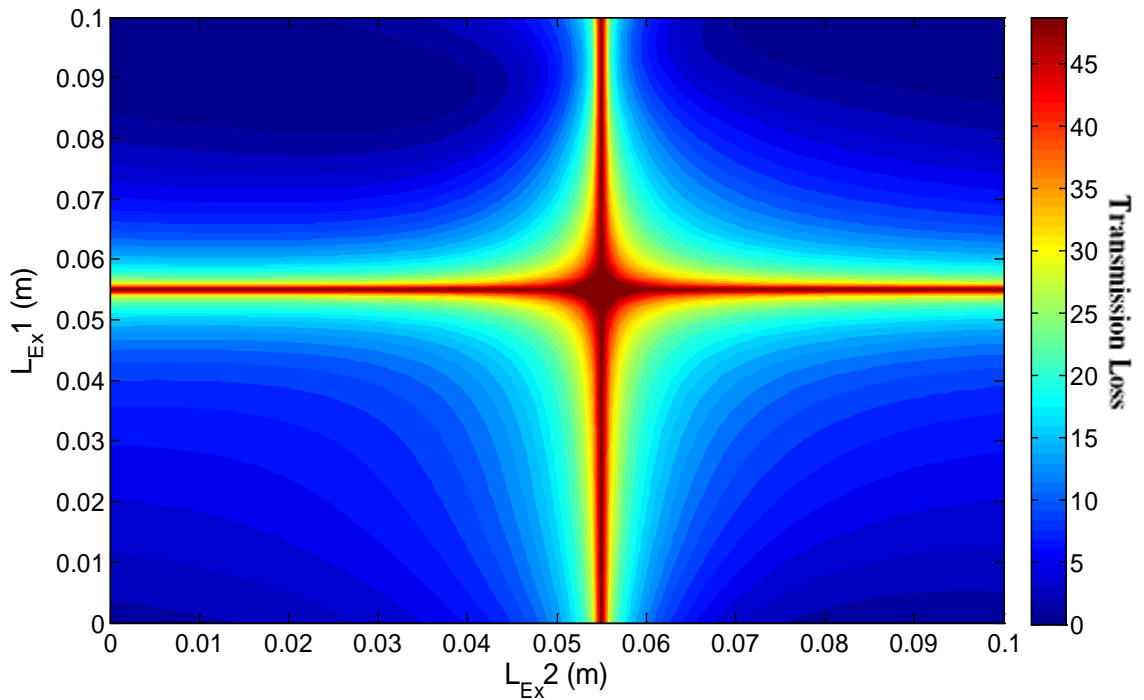


**Figure 4.7: Transmission loss of single expansion chamber silencer with extended tubes by changes outlet extended tube length.**

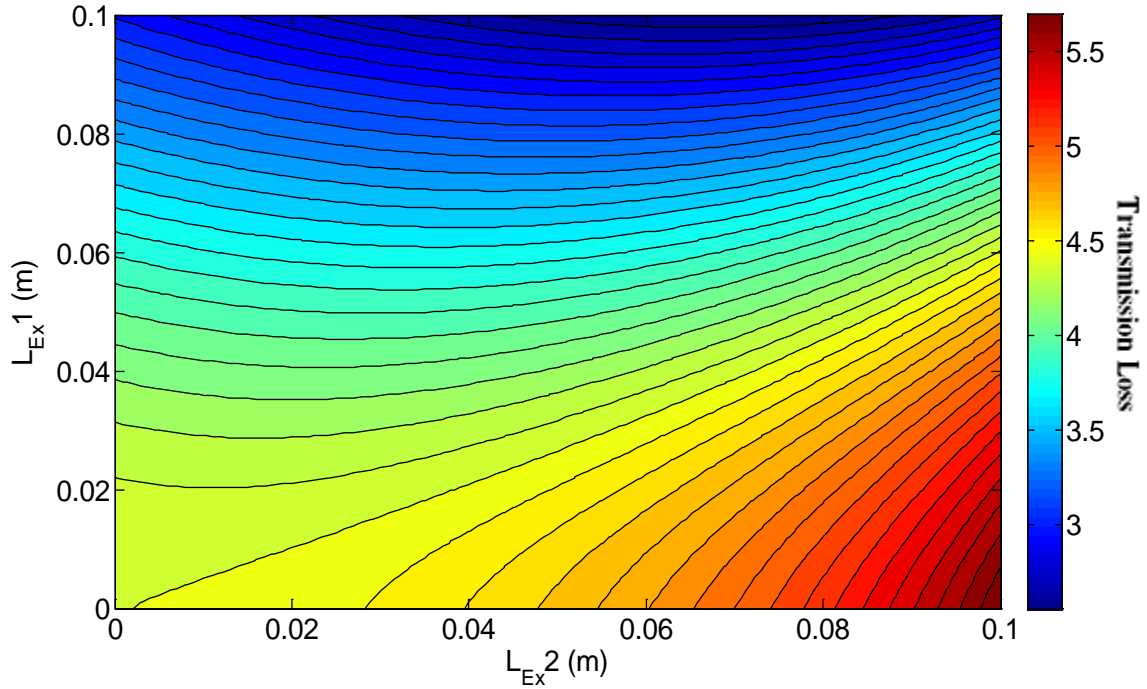
Figures 4.8 and 4.9 show the optimized inlet and outlet extended tube lengths which give best attenuation of noise waves having the frequencies 1500,412.5 Hz respectively.

If there is a noise wave with a frequency 1500 Hz, Figure 4.8 gives the optimized lengths of inlet and outlet extended tubes that give high transmission loss. So, if the inlet extended tubes length equals 0.06 m, the outlet extended tubes length should be 0.055 m to get best transmission loss.

If the wave frequency is 412.5 Hz and the outlet extended tubes length is 0.1 m, the length of the inlet extended tubes give maximum transmission loss is zero meter as indicates in Figure 4.9.



**Figure4.8:Extended tubes' lengths effect on TL of singleexpansion chamber silencer with extended tubes at frequency equal 1500 Hz.**



**Figure 4.9: Extended tubes' lengths effect on TL of single expansion chamber silencer with extended tubes at frequency equal 412.5 Hz.**

#### 4.4.2 Double Expansion chamber Silencer

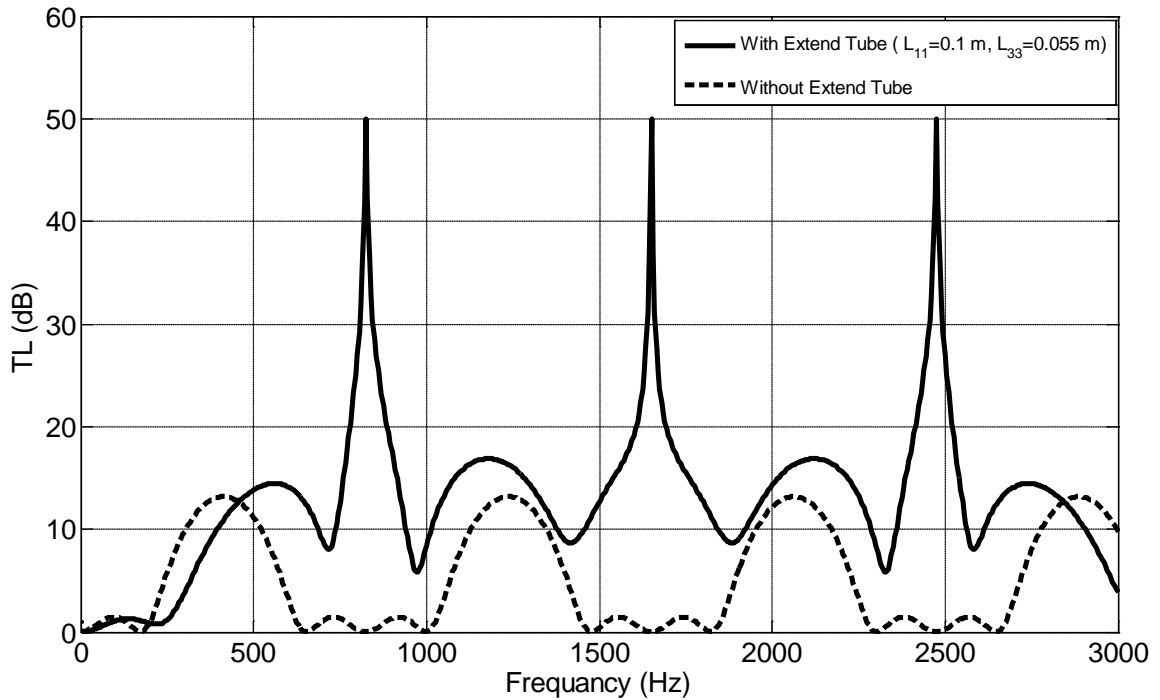
Table 4.3 shows geometrical parameters have been used to measure the transmission loss of silencer shown in Figure 4.3.

**Table 4.3: Geometrical parameters of double expansion chamber silencer with and without extended tube**

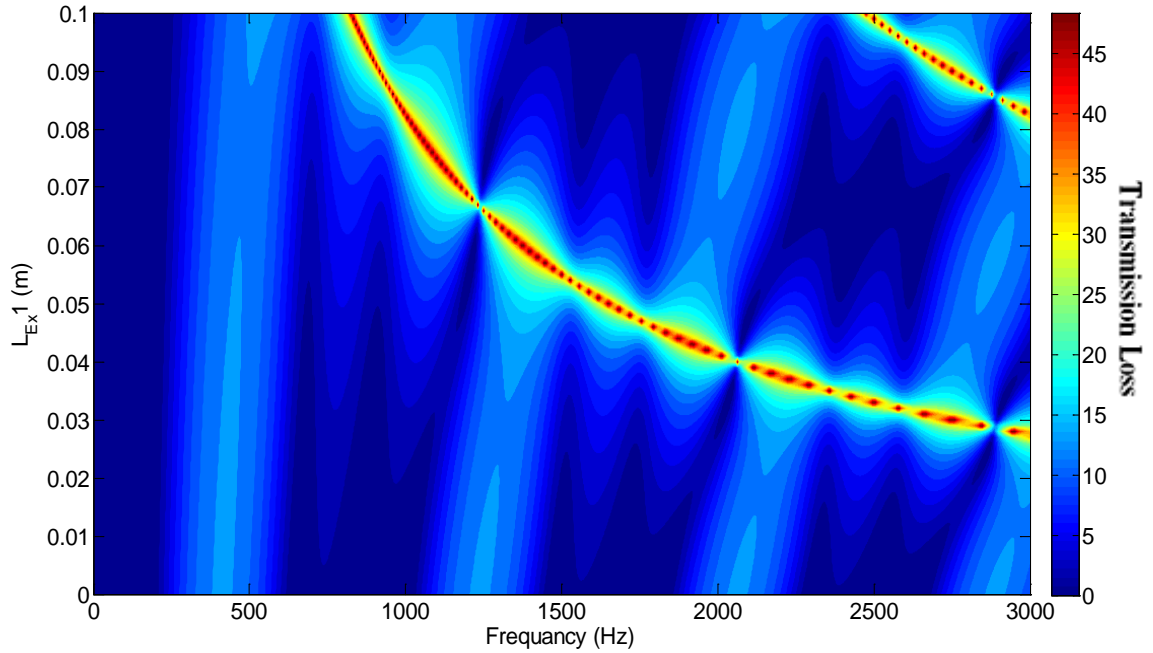
Silencer types	Expansion chambers' areas ( $m^2$ )	Ducts' areas ( $m^2$ )	Expansion chambers' lengths (m)	First extended tube length $L_{11}(m)$	Second extended tube length $L_{33}(m)$
With extended tubes	0.0942	0.0314	0.2	0.1	0.055
Without extended tubes	0.0942	0.0314	0.2	0	0

Figure 4.10 presents the difference of transmission loss between double expansion chamber silencer with and without extended tubes. Also, it is noted that the silencer with extended tubes has three wave major reflection with transmission loss reaches to 50 dB. So, the transmission loss of the silencer with extended tubes is better than one without extended tubes at all frequencies.

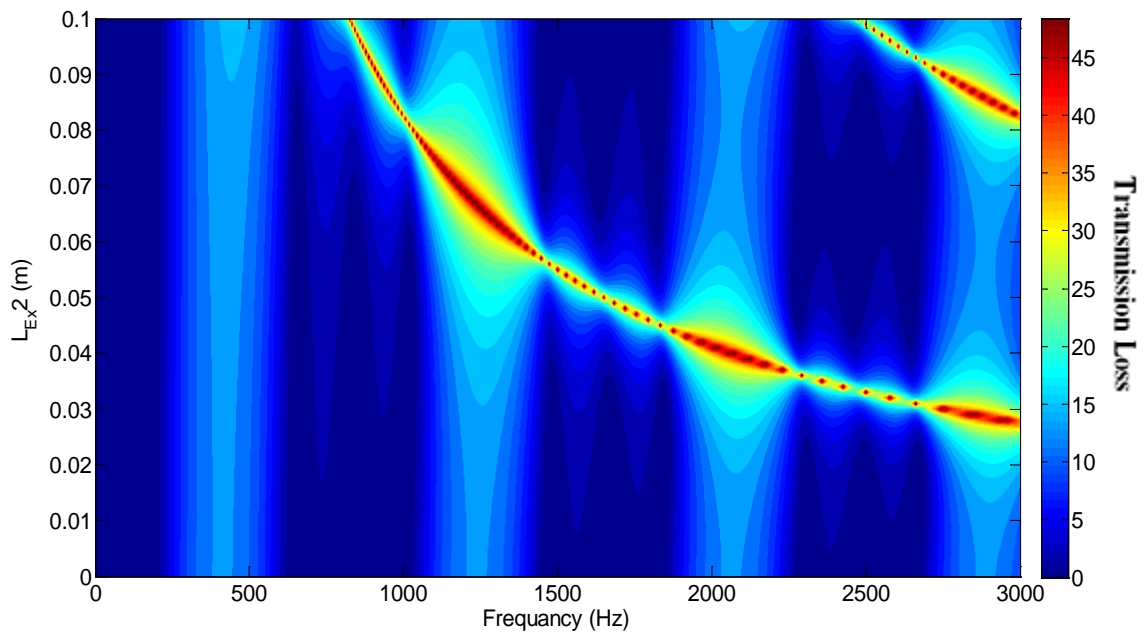
Figures 4.11 and 4.12 show three dimensional contour plots which generated using Equations 2.4 and 4.23 and it present the effect of inlet and outlet extended tubes lengths on transmission loss. From these figures, one can optimize the lengths of the extended tubes that give maximum transmission loss.



**Figure 4.10: TL of double expansion chamber silencer with and without extended tube.**



**Figure 4.11: Transmission loss of double expansion chamber silencer with extended tubes by changes inlet extended tube length.**



**Figure 4.12: Transmission loss of double expansion chamber silencer with extended tubes by changes outlet extended tube length.**

Table 4.4 shows the overall transmission loss of double expansion chambersilencer with and without extended tubes. The first silencer in the table has overall transmission loss lower than the second one. So, these results prove that it is better to use extended tubes in the double expansion chambersilencers.

**Table 4.4:OLT of double expansion chamber silencer.**

Silencers type depend on extended tubes		OLT (dB)
1	Without extended tubes	24.9258
2	With extended tubes	42.1540

#### 4.4.3 Triple Expansion chamber Silencer

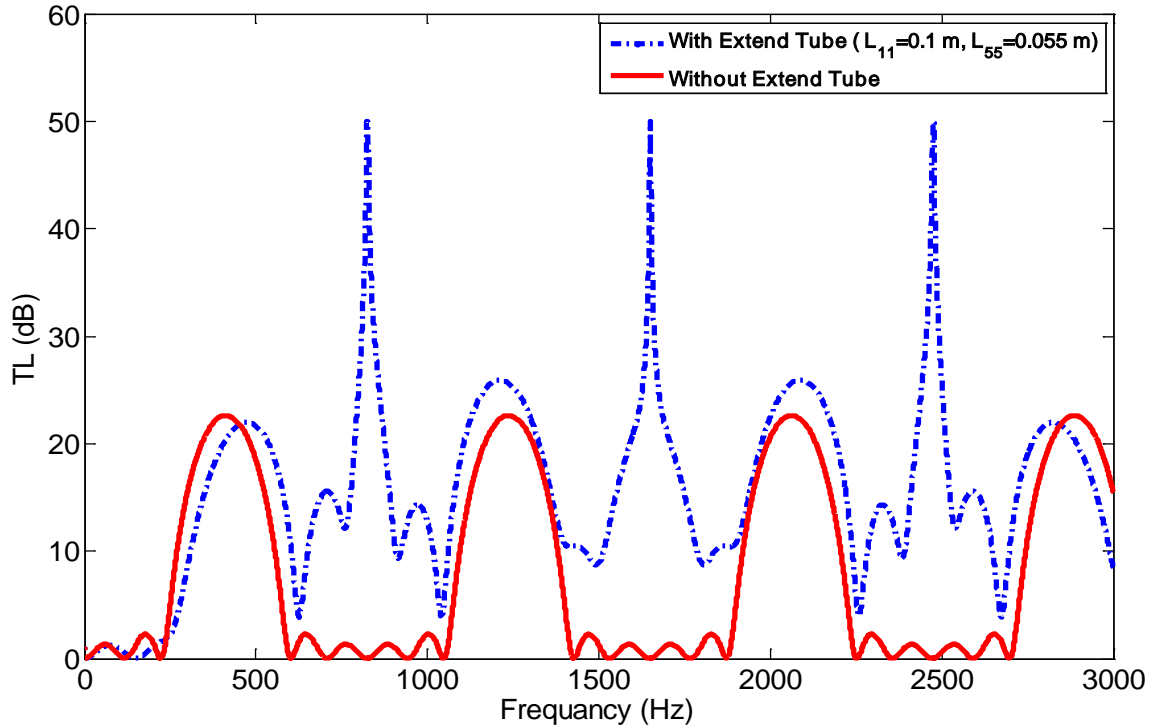
Table 4.5 shows the geometrical parameters of two configurations of triple expansion chamber silencer, as example, which used to get the transmission loss.

**Table 4.5:Geometrical parameters of triple expansion chamber silencer with and without extended tubes**

Silencer types	Expansion chambers' areas ( $m^2$ )	Ducts' areas ( $m^2$ )	Expansion chambers' lengths (m)	First extended tube length $L_{11}(m)$	Second extended tube length $L_{55}(m)$
With extended tubes	0.0942	0.0314	0.2	0.1	0.055
Without extended tubes	0.0942	0.0314	0.2	0	0

Figure 4.13 presents the difference of transmission loss between triple expansion chamber silencer with and without extended tubes. The transmission loss of the silencer with

extended tubes is higher than one without extended tubes at most frequencies, as in double expansion chamber silencer case.



**Figure 4.13: TL of triple expansion chambersilencer with and without extended tubes.**

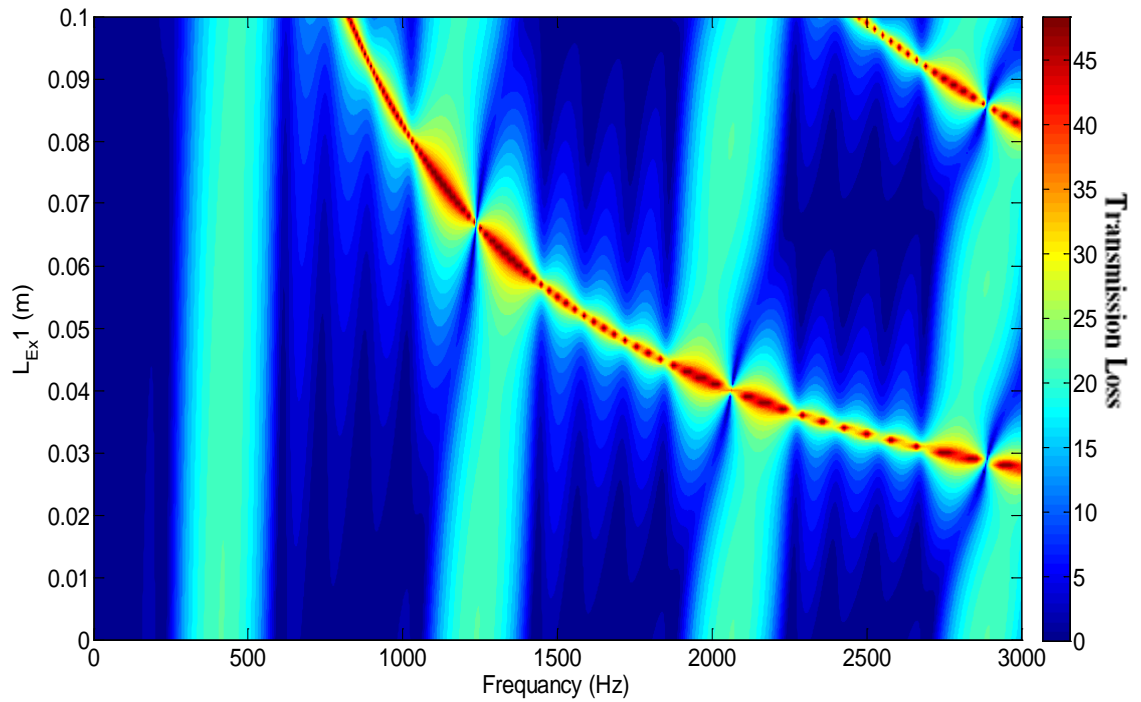
Table 4.6 shows the overall transmission loss of triple expansion chambersilencer with and without extended tubes. The silencer with extended tubes has the maximum overall transmission loss in Table 4.6 which means the optimum silencer is that one with extended tubes.



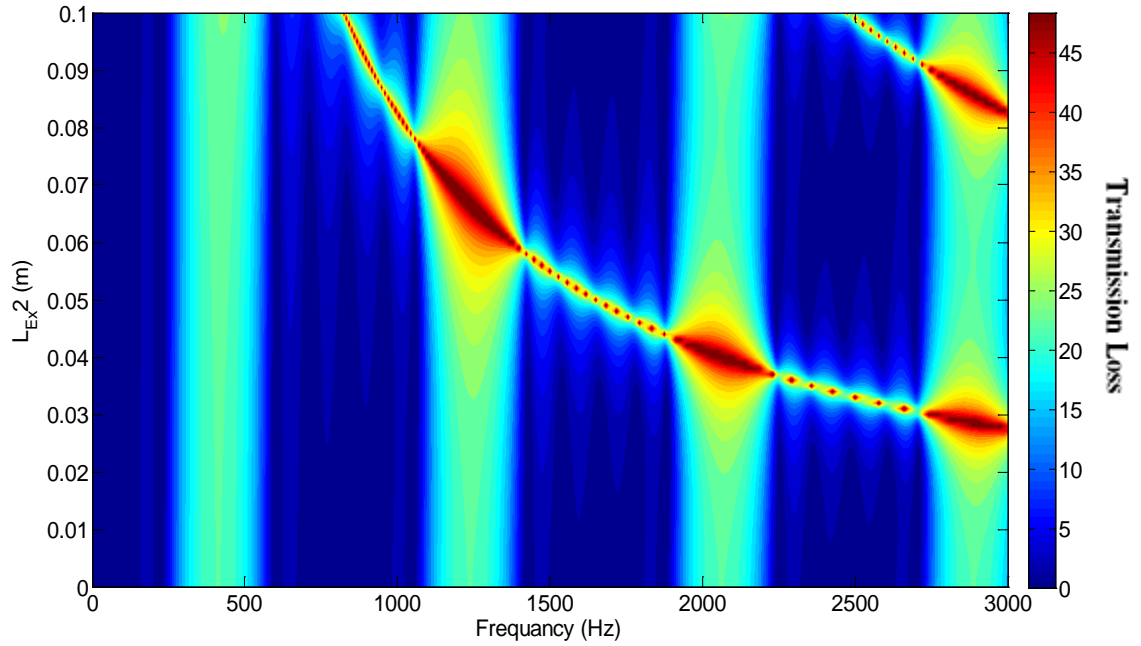
**Table 4.6: OLT of triple expansion chamber silencer.**

Silencers type depend on extended tubes		OLT (dB)
1	Without extended tubes	32.2966
2	With extended tubes	50.3303

Figure 4.14 and 4.15 generated using Equations 2.4 and 4.24. It presents two dimensional contour plot of transmission loss of triple expansion chamber silencer with extended tubes. From these figures one can get the optimum lengths of the extended tubes that give high transmission loss as in single and double expansion chamber silencer case.



**Figure 4.14: Transmission loss of triple expansion chamber silencer with extended tube by changes inlet extended tubes length.**



**Figure 4.15: Transmission loss of triple expansion chamber silencer with extended tube by changes outlet extended tubes length.**

# **CHAPTER 5**

## **ENHANCING ATTENUATION (II)**

### **APPLYING CHIRP AND TAPER FUNCTIONS**

The expansion chambersilencer geometry design is realized by utilizing modifications in cross sectional areas or modifications in the lengths of the expansion chambers and connecting tubes, or combined changes of both. It is beneficial to apply taper and chirp functions to the expansion chamber geometry which result in effects similar to the window functions utilized in the field of digital signal processing.

Hawwa[41] applied on the wall undulations, different type of taper functions to improve the design by widening the stopband and suppressing the passband ripple. He showed that the factors such as filter response of digital system processing filters can be utilized as a lead in shaping the desired filter response of periodic physical filters. Also he [42]presented an analysis of sound waves in a circular cylindrical duct having a geometric periodicity at its wall by applying a chirp functions on the wall periodicity. He showed that when the chirp function is chosen carefully then the change in the reflectivity spectrum can be favorable leading to blocking a wider range of frequencies.

## 5.1 CHIRP FUNCTIONS

The chirp function used here is inspired from signal chirp function which is a signal whose frequency increases and/or decreases with the time[43]. Applying this concept to the expansion chambers' lengths and connecting tubes' lengths, the value of the lengths can be increased and/or decreased depends on an equation from chirp functions. The expansion chambers and connecting tubes areas are constant. There are many types of chirp functions like:

- LinearlyIncreaseChirp Function.
- LinearlyDecreaseChirp Function.
- Uniform Chirp Function.
- Combination between first and second types.

### 5.1.1 Chirp Function Description

For linearly increase chirp function the lengths increases linearly as Equation 5.1 .

$$L(i) = L_0 + h\Delta L \quad (5.1)$$

Where  $L_0$  is the basic length,  $h = [0,1,2, \dots, n]$  is the expansion chamber or connecting tube number, and  $\Delta L$  is the chirp rate. See Figure 5.1(a)

The lengths of expansion chambers and connecting tubes with linearly decrease chirp function varies according Equation 5.2, see Figure 5.1(b)

$$L(i) = L_0 - h\Delta L \quad (5.2)$$

Figure 5.1(c) shows a triple expansion chamber with uniform chirp function. This means that all lengths of expansion chambers and connecting tubes are equal according to Equation 5.3.

$$L(i) = L_0 \quad (5.3)$$

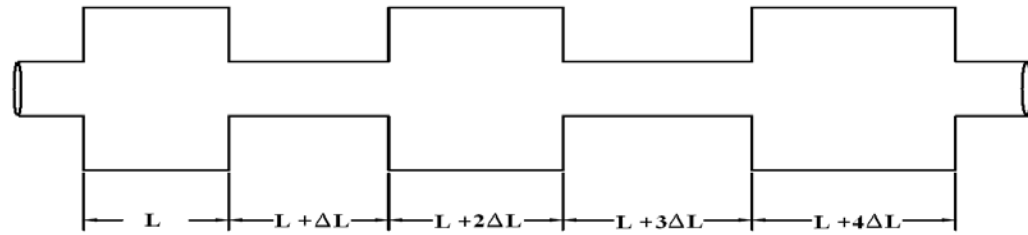
Figures 5.1(d) and 5.1(e) show graphical representation of triple expansion chamber by applying a combination of linearly increase and linearly decrease chirp functions, depend on Equations 5.4 and 5.5.

$$L(i) = \begin{cases} L_0 + h\Delta L & , h = 0,1,2 \\ L_0 + (4-h)\Delta L, & h = 3,4 \end{cases} \quad (5.4)$$

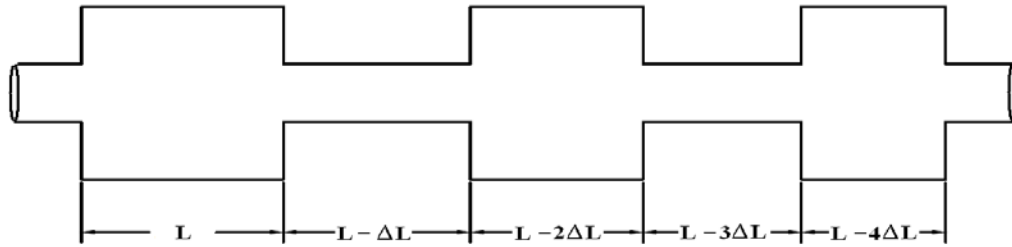
Equation 5.4 presents “increasing then decreasing” chirp function as Figure 5.1(d) shown.

Figure 5.1(e) and Equation 5.5 presents “decreasing then increasing” chirp function.

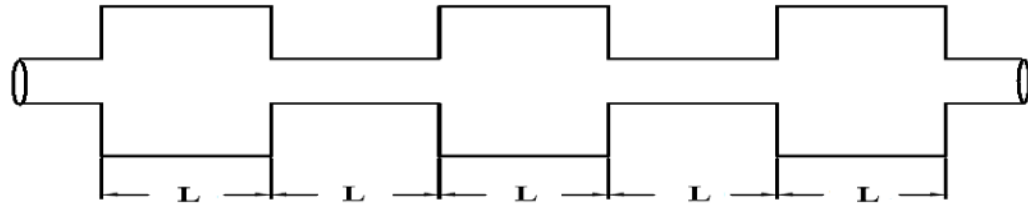
$$L(i) = \begin{cases} L_0 - h\Delta L & , h = 0,1,2 \\ L_0 - (4-h)\Delta L, & h = 3,4 \end{cases} \quad (5.5)$$



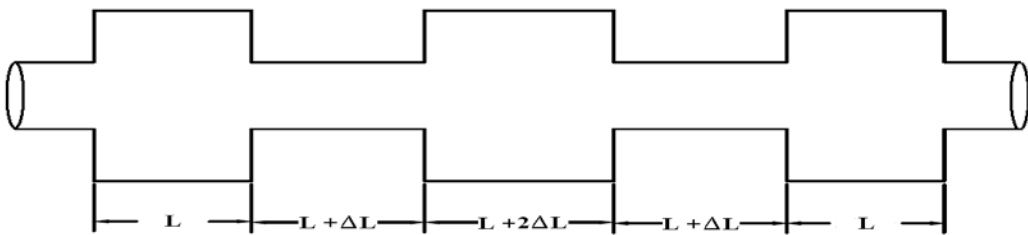
(a)



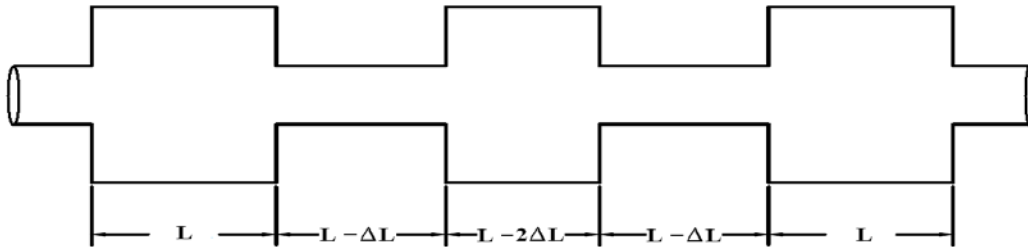
(b)



(c)



(d)



(e)

**Figure 5.1: Schematic of a silencer with triple expansion chamber using chirp functions: a) linearly increasing b) linearly decreasing c) uniform d) “increasing then decreasing” e) “decreasing then increasing”**

### 5.1.2 Chirp Function Results

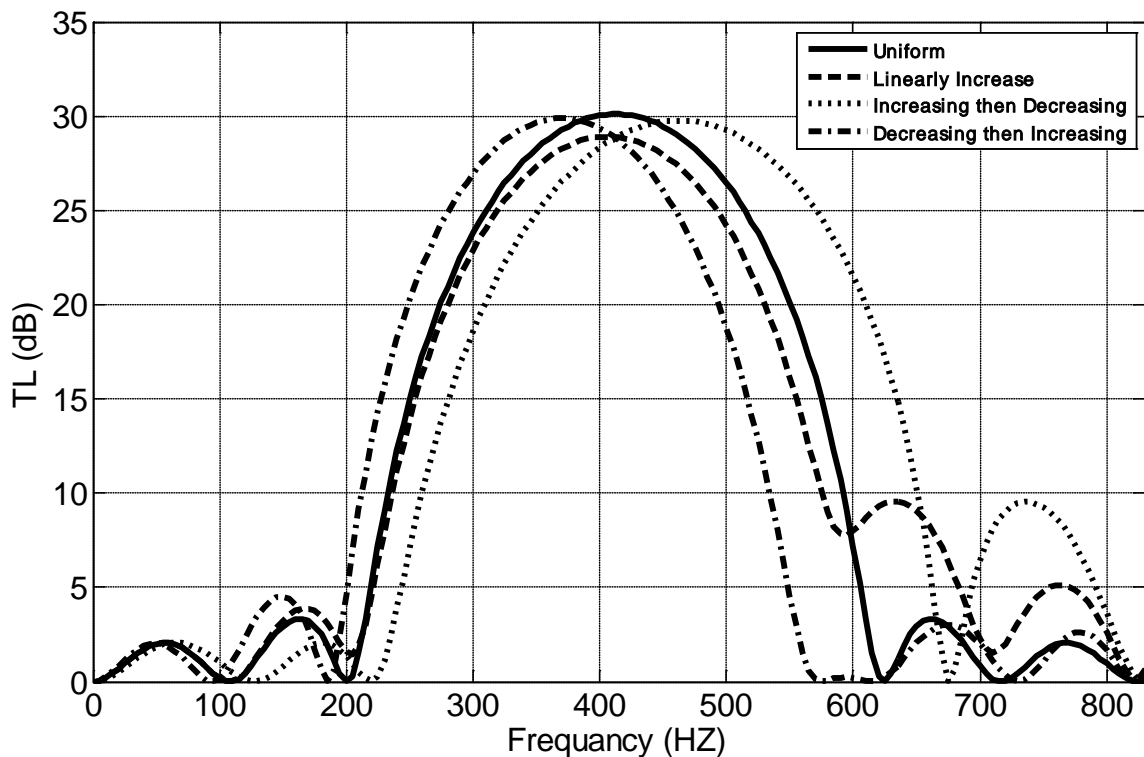
Table 5.1 shows the parametric geometry of triple expansion chamber silencers which used to study the effect of chirp functions on transmission loss.

**Table 5.1: Geometrical parameters of a triple expansion chamber silencer with chirp functions.**

Chirp function	First expansion chamber length (m)	First connecting tube length (m)	Second expansion chamber length (m)	Second connecting tube length (m)	Third expansion chamber length (m)	Duct radius (m)	Expansion chambers radius (m)	Connecting tubes radius (m)
Uniform	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.1
Linearly increase	0.16	0.18	0.2	0.22	0.24	0.1	0.2	0.1
Linearly decrease	0.24	0.22	0.2	0.18	0.16	0.1	0.2	0.1
Increasing then decreasing	0.16	0.18	0.2	0.18	0.16	0.1	0.2	0.1
Decreasing then increasing	0.24	0.22	0.2	0.22	0.24	0.1	0.2	0.1

Every chirp function has an effect on the transmission loss of triple expansion chamber silencer. Figure 5.2 shows the performance property (TL) of triple expansion chamber where the expansion chambers and connecting tubes lengths have been controlled by different chirp functions according to Figure 5.1 and Table 5.1. Solid line represents the transmission loss affected by applied uniform chirp function on triple

expansion chambersilencer lengths. The acoustic stopband starts from the frequency 201 Hz and ends at the frequency 624 Hz Hence, the width of the stopband is 423 Hz. The maximum transmission loss of 30.11 dB is observed to occur at the frequency 412.5 Hz which is the center frequency in the curve. The overall transmission loss of uniform chirp function curve is calculated to be 39.9307 dB.



**Figure 5.2: TL of triple expansion chamber silencer with different chirp functions.**

Dashed line represents the transmission loss of triple expansion chamber silencer shown in Figure 5.1(a). Acoustic stopband in this case starts from the frequency 205 Hz and ends at the frequency 594 Hz. Hence, the width of the stopband is 389 Hz. The maximum transmission loss is 28.92 dB which is observed to occur at the frequency 405 Hz which is



to the left of center frequency by 7.5 Hz. Also, the overall transmission loss is calculated to be 36.8016 dB. The transmission loss curve of the silencer shown in Figure 5.1(b) has same behavior of linearly increase chirp function curve.

This means transmission loss curve of linearly increase or linearly decrease chirp functions has low stopband width comparing with uniform chirp function. On other hand, the transmission loss curve of uniform function has side ripple with high transmission loss rather than linearly increase or linearly decrease chirp function curves especially on the right side.

Dotted line shows transmission loss properties of silencer shown in Figure 5.1(d). The acoustic stopband starts and ends from the frequencies 220 Hz, and 676 Hz respectively. Hence, the width of the stopband is 456 Hz. The maximum transmission loss of 29.67 dB is observed to occur at the frequency 462.5 Hz which is at the right of center frequency. The transmission loss curve of “increasing then decreasing” chirp function has two left side ripples with transmission loss less than 3 dB. Also, it has one right side ripple with almost 10 dB transmission loss. The overall transmission loss is 38.1985 dB.

Finally, the dash-dot line represents the transmission loss of the silencer shown in Figure 5.1(e). The acoustic band starts from the frequency 186 Hz and ends at the frequency 575 Hz. With stopband width equals 389 Hz. The maximum transmission loss of 29.89 dB occurs when the frequency equals 370 Hz which is at the left of center frequency. The transmission loss curve of “decreasing then increasing” chirp function has two left side ripples with transmission loss higher than “increasing then decreasing” chirp function

curve which is around 4 dB. But, in the right side it has three ripples with transmission loss less than 4dB. The overall transmission loss is calculated to be 38.6038 dB.

**Table 5.2: Comparison between different triple expansion chambers silencers.**

Silencers type depend on chirp function		OLT (dB)	Stopband width (Hz)
1	Uniform	39.9307	423
2	Linearly Increase	36.8016	389
4	Increase then Decrease	38.1985	456
5	Decrease then Increase	38.6038	389

Table 5.2 shows a comparison between different expansion chambers silencers depending on the overall transmission loss (OLT), and the stopband width.

The “increasing then decreasing” silencer configuration is the optimum silencer because it has the maximum stopband width. Also, uniform silencer configuration is the optimum one between all silencers in Table 5.2 depends on the maximum overall transmission loss concept. So, the noise frequencies that need to be attenuated can be controlled using chirp functions.

## 5.2 TAPER FUNCTIONS

In signal processing, a window function (Taper function) is a mathematical function which has a value equals zero outside of some chosen interval[44].

There are many types of tapering functions. These include:

- Triangular window
- Linearly increase window
- Linearly decrease window
- “Decreasing then increasing” window
- Uniform window

### 5.2.1 Taper Function Description

Each function has an equation as following:

Triangular window equation is

$$w(h) = \frac{2}{N-1} \left( \frac{N-1}{2} - \left| h - \frac{N-1}{2} \right| \right) \quad (5.6)$$

Where  $N$  is the number of the silencer expansion chamber plus two,  $h$  is the expansion chamber number (1, 2, ...,  $N-2$ ).

Linearly increase window equation is

$$w(h) = h \quad (5.7)$$

Linearly decrease window equation is

$$w(h) = N - (h + 1) \quad (5.8)$$

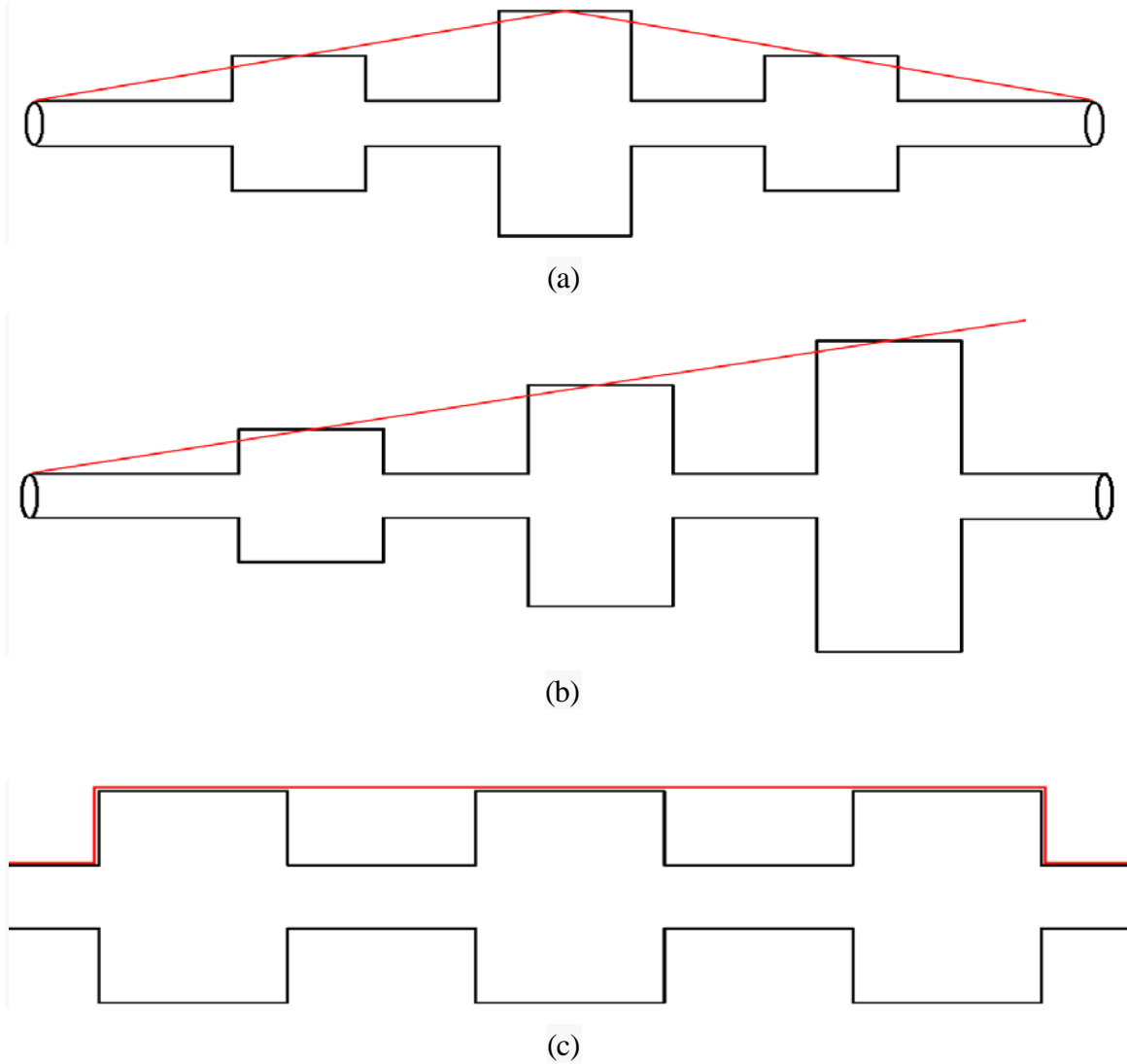
Uniform window equation is

$$w(h) = 1 \quad (5.9)$$

A “decreasing then increasing” window equation is a combination of linearly increase and linearly decrease window equations.

Optimized designs are realized by utilizing changes in cross sectional areas of the expansion chambers. The light line in Figures 5.3 shows some of taper function graphical

representations for triple expansion chamber silencer which it used to control silencer areas.



**Figure 5.3:Some of taper windowgraphical representation for triple expansion chamber silencer: (a) Triangular (b) Linearly increase (c) Uniform**

### 5.2.2 Taper Function Results

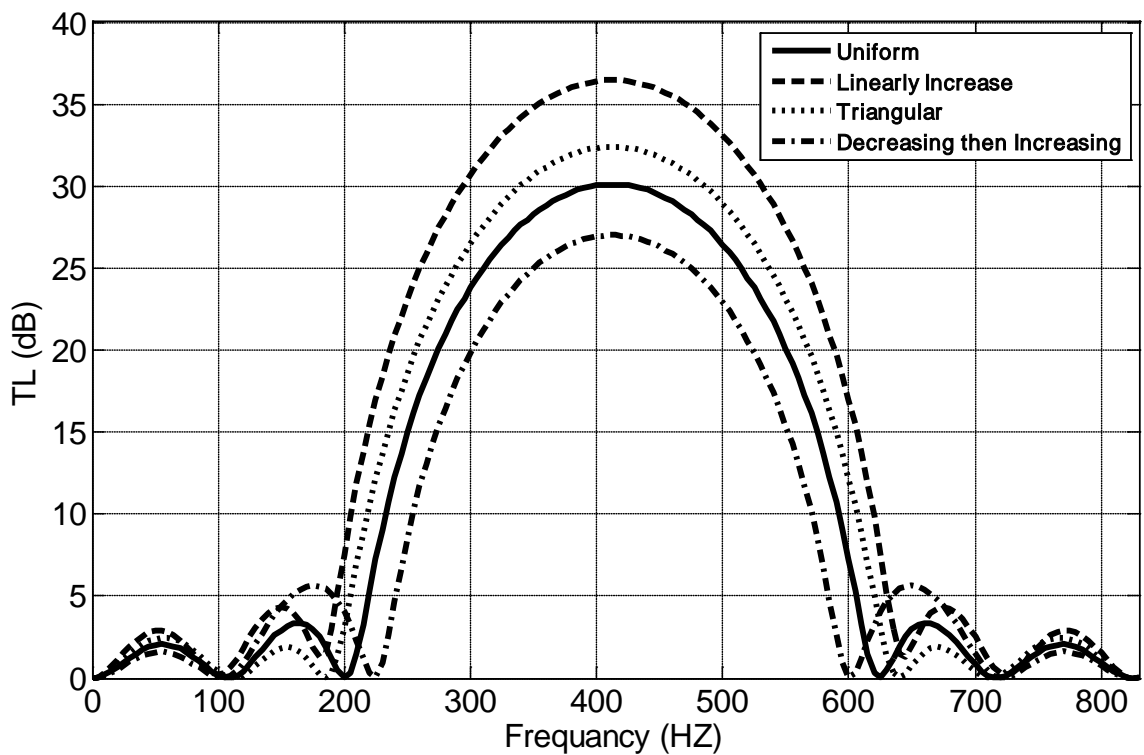
Table 5.3 shows geometrical parameters that used to study transmission loss of triple expansion chamber silencer with different taper functions.

**Table 5.3: Geometrical parameters of triple expansion chambersilencers with taper functions**

Taper function	First expansion chamber area (m <sup>2</sup> )	Second expansion chamber area(m <sup>2</sup> )	Third expansion chamber area (m <sup>2</sup> )	Duct area (m <sup>2</sup> )	Connecting tubes area (m <sup>2</sup> )	connecting tubes lengths (m)	Expansion chambers lengths (m)
Uniform	<b>0.1257</b>	0.1257	0.1257	0.0314	0.0314	0.2	0.2
Linearly increase	<b>0.1257</b>	0.1634	0.2011	0.0314	0.0314	0.2	0.2
Linearly decrease	0.2011	0.1634	<b>0.1257</b>	0.0314	0.0314	0.2	0.2
Triangle	<b>0.1257</b>	0.1634	0.1257	0.0314	0.0314	0.2	0.2
Decreasing then increasing	<b>0.1257</b>	0.0880	0.1257	0.0314	0.0314	0.2	0.2

Every taper function has an effect on the transmission loss of triple expansion chambersilencer. Figure 5.4 shows the performance property (TL) of triple expansion chamber where the expansion chambers' areas have been controlled by different taper functions according to Figure 5.3 and Table 5.3. Dashed line shows the transmission loss silencer shown in Figure 5.3(b). The acoustic stop band starts from the frequency 182 Hz and ends at the frequency 644 Hz. Hence, the width of the stop band equals 462 Hz. The maximum transmission loss of 36.46 dB is observed to occur at the frequency 412.5 Hz which is the

center frequency in the curve. The overall transmission loss of linearly increase taper function curve is 46.465dB. Below the frequency 182 Hz the transmission loss curve has two left side ripples with amplitude less than 4.5 dB. Also, it has two right side ripples with same amplitude above the frequency 672 Hz. The effect of linearly decrease taper function on the transmission loss of triple expansion chambersilencer is the similar to the effect of linearly increase taper function.



**Figure 5.4: TL of triple expansion chamber silencer with different taper functions.**

Dotted line represents the transmission loss of a triple expansion chamber silencer shown in Figure 5.3 (a). The maximum transmission loss decreases to 32.38 dB which occurs at the same center frequency 412.5 Hz. Acoustic stopband in this case starts from the frequency 185.5 Hz and ends at the frequency 640 Hz. This means that the width of the stopband is 454.5 Hz. Below the frequency 185.5 Hz the transmission loss curve has two

left side ripples with amplitudes less than 2 dB. Above the frequency 640 Hz there are two right side ripples with amplitudes less than 2 dB. The overall transmission loss is calculated to be 42.3337 dB.

Solid line shows transmission loss properties of triple expansion chambersilencer shown in Figure 5.3(c). The acoustic stopband starts and ends at the frequencies 201.5 Hz and 625 Hz respectively with stopband width equals 423.5 Hz. 30.11 dB is the maximum transmission loss occurs at the center frequency 412.5 Hz. The transmission loss curve of uniform taper function has two left side ripples and two right side ripples with transmission loss less than 3.5 dB. Also, the overall transmission loss is 39.9307 dB which is less than the overall transmission loss of linearly increase and triangular taper functions cases.

Last curve which is the dash-dot line represents the transmission loss affected by applied “decreasing then increasing” taper function on triple expansion chambersilencer. The acoustics band has a maximum transmission loss equals 27.01 dB at the frequency 412.5 Hz and the acoustic band starts at the frequency 224 Hz and ends at the frequency 601 Hz. Hence, the width of the stopband is calculated to be 377 Hz which is the lowest stopband width in Figure 5.4. At each side of the curve has two ripples with transmission loss less than 5.644 dB which is the highest ripple amplitude in Figure 5.4. Finally, the overall transmission loss is 36.6231 dB.

All spectra in Figure 5.4 have symmetrical behavior around center frequency.

**Table 5.4: Comparison between different silencers with taper functions.**

Silencers type depend on taper functions		OLT (dB)	Stopband width (Hz)	Maximum TL (dB)
1	Linear Increasing	46.4650	462	36.46
2	Triangular	42.3337	454.5	32.38
3	Uniform	39.9307	423.5	30.11
4	Decrease then Increase	36.6231	337	27.01

Table 5.4 shows a comparison between four cases of triple expansion chambers silencers with taper functions according to following parameters:

- Overall transmission loss (OLT).
- Maximum amplitude.
- Stopband width.

Linearly increase silencer configuration has the maximum overall transmission loss and maximum transmission loss at the frequency 412.5 Hz. Also, it has the highest stopband width between all silencers as Table 5.4 shows. So, the optimum choice used to attenuate noise wave would be linearly increase triple expansion chambers silencer configuration.

### **5.3 OPTIMUM CHIRP AND TAPER TRIPLE EXPANSION CHAMBERSILENCER**

The ideal solution of triple expansion chambers silencer configuration, gives best transmission loss results according to previous analysis. Best taper and chirp

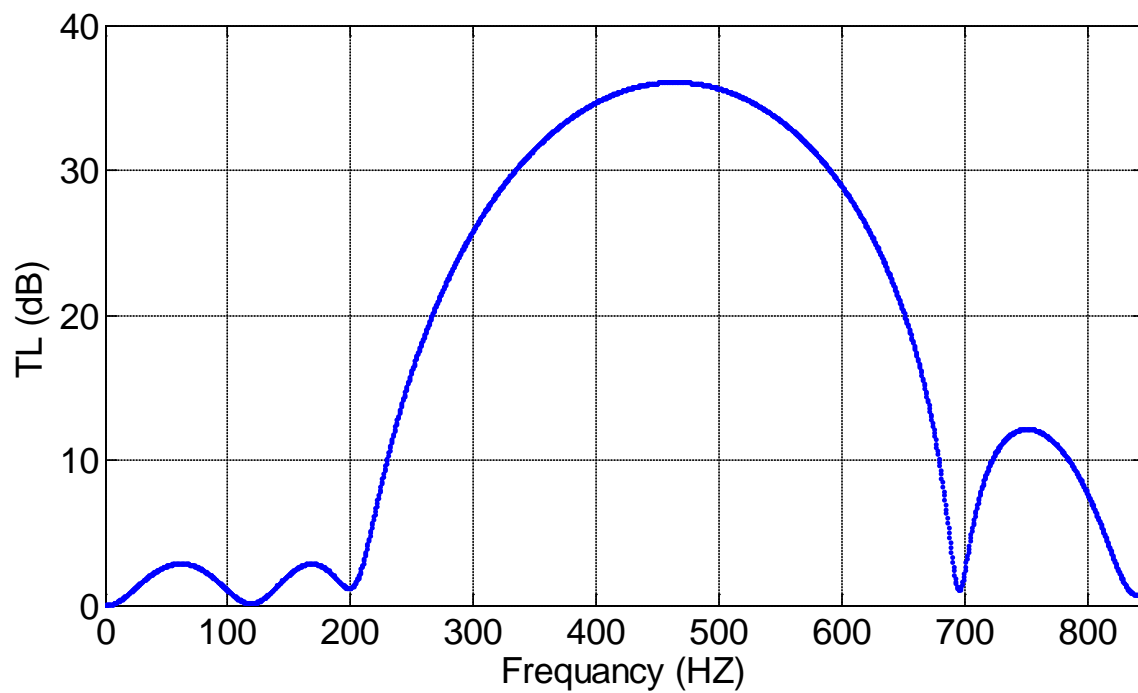


functions used to control the geometry of triple expansion chamber silencer to come up with silencer that has best transmission loss results.

Best transmission loss curve has high amplitude and high stopband width. “Increasing then decreasing” chirp function silencer configuration has the maximum stopband width and linearly increase taper silencer configuration gives highest overall transmission loss according to previous analysis.

Combine “increasing then decreasing” chirp function properties with linearly increase taper function properties to come up with best silencer has highest transmission loss properties.

Figure 5.5 presents the transmission loss affected by linearly increase taper function and “increasing then decreasing” chirp function on triple expansion chamber silencer. The acoustics band has a maximum transmission loss equals 36.11 dB at the frequency 463 Hz and it starts from the frequency 198 Hz and ends at the frequency 695 Hz. Hence, the width of the stopband is 497 Hz which is the maximum stopband width obtained. The transmission loss curve has two left side ripples with transmission loss less than 2.9 dB. Also, it has one right side ripple with almost 12.2 dB transmission loss. Finally, the overall transmission loss of new silencer is 44.6978 dB which is almost similar to the maximum overall transmission loss got in previous study.



**Figure 5.5: TL of triple expansion chamber silencer with “increase then decrease” chirp function and linearly increase taper function.**

# CHAPTER 6

## CONCLUSIONS AND RECOMMENDATIONS

This study focuses on the effectiveness of silencers with multiple expansion chambers in reducing acoustic noise.

Enhanced designs are realized by utilizing changes in cross sectional areas and changes in the lengths of the expansion chambers as well as combined changes of both. Enhanced designs are also achieved if the expansion chambers are supplemented by extended tubes.

After this study we conclude the following:

- 1) Double expansion chamber silencers show overall transmission loss higher than single expansion chamber silencer. Also, triple expansion chamber silencer gives high overall transmission loss, more than single and the double expansion chamber silencers. This means that it is better to use multiple expansion chamber silencer than single ones for acoustic wave attenuation.
- 2) Silencers having expansion chambers with extended tubes give more attenuation than silencers having expansion chambers without extended tubes.
- 3) In triple expansion chamber silencer, the transmission loss value and the noise frequency stopband are controlled by chirp functions applied to the expansion chambers lengths. Controlling the stopband location within the frequency

spectrum is obtained by making a right or left shift to the center frequency depending on the chirp functions used.

- 4) In triple expansion chamber silencers, the transmission loss is affected by taper functions applied to the expansion chamber volumes. So, the transmission loss can be controlled using taper functions. Transmission loss is increased or decreased depending on the taper function used.

This study tends to provide general guidelines for enhancing the acoustic attenuation behavior of silencers with expansion chambers.

Future work built on this study can include the application of multi-dimensional optimization techniques for designing improved silencers. Also, study the effectiveness of nonlinear extended tubes on the transmission loss for expansion chamber silencers.

## NOMENCLATURE

$TMM$	Transfer matrix method
$OLT$	Overall transmission loss
$A_j$	Area
$L_j$	Length
$P$	Pressure
$k$	Wave number
$f$	Frequency
$\omega$	Angular velocity
$\lambda$	Wave length
$c$	Sound speed
$x$	Distance
$\rho_o$	Air density
$U$	Volume velocity
$IL$	Insertion loss
$NR$	Noise reduction
$TL$	Transmission loss
$T_i$	Transmitted pressure amplitude in expansion chamber silencer
$R_i$	Reflected pressure amplitude in expansion chamber silencer

$B_i$	Transmitted wave pressure amplitude in expansion chamber silencer with extended tube
$D_i$	Reflected wave pressure amplitude in expansion chamber silencer with extended tube
$t$	Time
$i$	$\sqrt{-1}$
$e$	Exponential function
$m$	First area ratio
$n$	Second area ratio
$g$	Third area ratio
$q$	Counting number
$N$	Expansion chamber number + 2
$Z_i$	Impedance

### **Subscripts**

$T_i$	Transmitted wave number
$R_i$	Reflected wave number
$I_i$	Incident wave number

## REFERENCES

- [1] [http://wiki.answers.com/Q/What\\_is\\_acoustics](http://wiki.answers.com/Q/What_is_acoustics).
- [2] [http://www.physics.byu.edu/research/acoustics/what\\_is\\_acoustics.aspx](http://www.physics.byu.edu/research/acoustics/what_is_acoustics.aspx).
- [3] A. L. Rogers, J. F. Manwell, and S. Wright, "Wind turbine acoustic noise," White Paper, Renewable Energy Research Laboratory, Department of Mechanical & Industrial Engineering, University Massachusetts at Amherst, MA, vol. 1003, 2002.
- [4] [http://www.engineeringtoolbox.com/noise-reduction-silencers-d\\_81.html](http://www.engineeringtoolbox.com/noise-reduction-silencers-d_81.html).
- [5] [http://www.acoustiquepn.ca/english/silencers.htm#Dispersive silencer](http://www.acoustiquepn.ca/english/silencers.htm#Dispersive%20silencer).
- [6] [http://pdf.directindustry.com/pdf/alde/active-silencer-for-circular-ductwork-systems/20636-184927-\\_2.html](http://pdf.directindustry.com/pdf/alde/active-silencer-for-circular-ductwork-systems/20636-184927-_2.html).
- [7] J. Lamancusa, "The transmission loss of double expansion chamber mufflers with unequal size chambers," *Applied Acoustics*, vol. 24, pp. 15-32, 1988.
- [8] A. Selamet and P. Radavich, "The effect of length on the acoustic attenuation performance of concentric expansion chambers: an analytical, computational and experimental investigation" *Journal of sound and vibration*, vol. 201, pp. 407-426, 1997.
- [9] A. Selamet and Z. Ji, "Acoustic attenuation performance of circular expansion chambers with offset inlet/outlet: I. Analytical approach," *Journal of sound and vibration*, vol. 213, pp. 601-617, 1998.
- [10] Z. Tao and A. F. Seybert, "A review of current techniques for measuring muffler transmission loss," 2003.

- [11] D. Suwandi, J. Middelberg, K. Byrne, and N. Kessissoglou, "Predicting the acoustic performance of mufflers using transmission line theory."
- [12] S. Gerges, R. Jordan, F. Thieme, J. Bento Coelho, and J. Arenas, "Muffler modeling by transfer matrix method and experimental verification," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 27, pp. 132-140, 2005.
- [13] Z. Ji, "Acoustic attenuation performance analysis of multi-chamber reactive silencers," *Journal of sound and vibration*, vol. 283, pp. 459-466, 2005.
- [14] R. Barbieri and N. Barbieri, "Finite element acoustic simulation based shape optimization of a muffler," *Applied Acoustics*, vol. 67, pp. 346-357, 2006.
- [15] C. Wu, X. Wang, and H. Tang, "Transmission loss prediction on SIDO and DISO expansion-chamber mufflers with rectangular section by using the collocation approach," *International journal of mechanical sciences*, vol. 49, pp. 872-877, 2007.
- [16] C. Wu, X. Wang, and H. Tang, "Transmission loss prediction on a single-inlet/double-outlet cylindrical expansion-chamber muffler by using the modal meshing approach," *Applied Acoustics*, vol. 69, pp. 173-178, 2008.
- [17] P. k. Florent masson, gonzalo herrera, "optimization of muffler transmission loss by using micro perforated panels," *VI congreso iberoamericano de acustica - FIA* 2008, 7/11/2008 2008.
- [18] K. S. Andersen, "Analyzing muffler performance using the transfer matrix method."



- [19] Z. Kang and Z. Ji, "Acoustic length correction of duct extension into a cylindrical chamber," *Journal of sound and vibration*, vol. 310, pp. 782-791, 2008.
- [20] B. Venkatesham, M. Tiwari, and M. Munjal, "Transmission loss analysis of rectangular expansion chamber with arbitrary location of inlet/outlet by means of Green's functions," *Journal of sound and vibration*, vol. 323, pp. 1032-1044, 2009.
- [21] D. Y. Maa, "Potential of microperforated panel absorber," *The Journal of the Acoustical Society of America*, vol. 104, p. 2861, 1998.
- [22] T. Wu, P. Zhang, and C. Cheng, "Boundary element analysis of mufflers with an improved method for deriving the four-pole parameters," *Journal of sound and vibration*, vol. 217, pp. 767-779, 1998.
- [23] J. T. Kim and J. G. Ih, "Transfer matrix of curved duct bends and sound attenuation in curved expansion chambers," *Applied Acoustics*, vol. 56, pp. 297-309, 1999.
- [24] A. Selamet, I. Lee, and N. Huff, "Acoustic attenuation of hybrid silencers," *Journal of sound and vibration*, vol. 262, pp. 509-527, 2003.
- [25] P. Joseph, C. Morfey, and C. Lowis, "Multi-mode sound transmission in ducts with flow," *Journal of sound and vibration*, vol. 264, pp. 523-544, 2003.
- [26] E. Dokumaci, "Effect of sheared grazing mean flow on acoustic transmission in perforated pipe mufflers," *Journal of sound and vibration*, vol. 283, pp. 645-663, 2005.
- [27] T. Kar and M. Munjal, "Generalized analysis of a muffler with any number of interacting ducts," *Journal of sound and vibration*, vol. 285, pp. 585-596, 2005.

- [28] S. Zhao, J. Wang, and Y. He, "Expansion-chamber muffler for impulse noise of pneumatic frictional clutch and brake in mechanical presses," *Applied Acoustics*, vol. 67, pp. 580-594, 2006.
- [29] H. Rammal and H. Boden, "Modified multi-load method for nonlinear source characterisation," *Journal of sound and vibration*, vol. 299, pp. 1094-1113, 2007.
- [30] A. Broatch, J. Serrano, F. Arnau, and D. Moya, "Time-domain computation of muffler frequency response: Comparison of different numerical schemes," *Journal of sound and vibration*, vol. 305, pp. 333-347, 2007.
- [31] F. Denia, A. Selamet, F. Fuenmayor, and R. Kirby, "Acoustic attenuation performance of perforated dissipative mufflers with empty inlet/outlet extensions," *Journal of sound and vibration*, vol. 302, pp. 1000-1017, 2007.
- [32] Y. Wang, H. He, and A. Geng, "Comparison and application of the experimental methods for multi-layer prediction of acoustical properties of noise control materials in standing wave-duct systems," *Applied Acoustics*, vol. 69, pp. 847-857, 2008.
- [33] M. C. Chiu, "Shape optimization of multi-chamber mufflers with plug-inlet tube on a venting process by genetic algorithms," *Applied Acoustics*, vol. 71, pp. 495-505, 2010.
- [34] M. C. Chiu and Y. C. Chang, "Shape optimization of multi-chamber cross-flow mufflers by SA optimization," *Journal of sound and vibration*, vol. 312, pp. 526-550, 2008.

- [35] M. C. Chiu and Y. C. Chang, "Numerical assessment of a space-constrained venting system with multi-chamber plug mufflers by GA method," *Journal of Marine Science and Technology*, vol. 18, pp. 317-332, 2010.
- [36] Z. Ji, "Boundary element acoustic analysis of hybrid expansion chamber silencers with perforated facing," *Engineering Analysis with Boundary Elements*, vol. 34, pp. 690-696, 2010.
- [37] P. Chaitanya and M. Munjal, "Effect of wall thickness on the end corrections of the extended inlet and outlet of a double-tuned expansion chamber," *Applied Acoustics*, 2010.
- [38] A. Mimani and M. Munjal, "Transverse plane wave analysis of short elliptical chamber mufflers: An analytical approach," *Journal of sound and vibration*, 2010.
- [39] L. H. Bell and D. H. Bell, *Industrial noise control: Fundamentals and applications* vol. 88: CRC, 1994.
- [40] M. L. Munjal, *Acoustics of ducts and mufflers with application to exhaust and ventilation system design*: Wiley-Interscience, 1987.
- [41] M. Hawwa, "Waveguides with periodic undulations inspired by dsp windowing," *Journal of sound and vibration*, vol. 209, pp. 699-706, 1998.
- [42] M. A. Hawwa, "Acoustic wave blocking in a duct with a chirped periodic wall," *Arabian Journal for Science and Engineering*, vol. 29, pp. 113-124, 2004.
- [43] <http://en.wikipedia.org/wiki/Chirp>.
- [44] [http://en.wikipedia.org/wiki/Window\\_function](http://en.wikipedia.org/wiki/Window_function).

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